Related Rates Day 1 Homework

1. Given $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = -4 \text{ cm/sec}$

Find $\frac{dV}{dt}$ when $r = 3$

$\frac{dV}{dt} = \frac{4}{3}(3)\pi r^2 \frac{dr}{dt}$ (remember chain Rule)

$\frac{dV}{dt} = 4(3)^2(-4)\pi$

$= -144\pi \text{ cm}^3/\text{sec}$

$\boxed{-144\pi \text{ cm}^3/\text{sec}}$

2. Given $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = -\frac{256\pi}{3}$

Find $\frac{dr}{dt}$ when $r = 8$

$\frac{dV}{dt} = \frac{4}{3}(3)\pi r^2 \frac{dr}{dt}$ (chain rule)

$\frac{-256\pi}{3} = 4\pi(8)^2 \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = -\frac{1}{3} \text{ cm/sec}}$

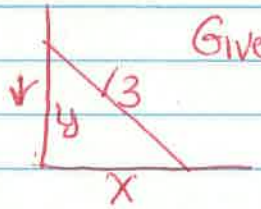
3. Given $\frac{dr}{dt} = 9 \text{ cm/min}$ $\quad A = \pi r^2$

Find $\frac{dA}{dt}$ when $r = 12$ cm

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ (chain rule)

$\frac{dA}{dt} = 2\pi(12)(9)$

$\boxed{= 216\pi \text{ cm}^2/\text{sec}}$

4. 

Given: $\frac{dy}{dt} = -7 \text{ ft/sec}$

Find $\frac{dx}{dt}$ when $x = 12$ ft

Use Pythagorean Theorem

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

Cancel all the two's to make life easier

Remember $\frac{dz}{dt} = 0$ b/c the length of the ladder is a constant

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

Use Pythagorean Theorem to find y

$12^2 + y^2 = 13^2$

$y^2 = 25$

$y = 5$

$12\frac{dx}{dt} + 5(-7) = 0$

$\frac{dx}{dt} = \frac{35}{12}$ ft/sec

5. Given $\frac{dr}{dt} = 2 m/min$ $\quad A = \pi r^2$

Find $\frac{dA}{dt}$ when $r = 13m$

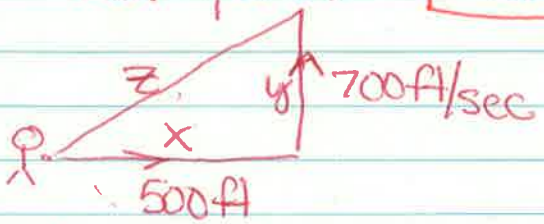
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ (use chain rule)

$= 2\pi(13)(2)$

$= 52\pi m^2/min$

$52\pi \ m^2/min$

6.



Given $\frac{dy}{dt} = 700 ft/sec$ $\quad x = 500$ (constant)

Find $\frac{dz}{dt}$ when $y = 1200ft = 1200ft$

Use Pythagorean Theorem

$dx/dt = 0$ Because x is not changing

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

Cancel all of the 2's

$y\frac{dy}{dt} = z\frac{dz}{dt}$

Find z using Pytagorean Theorem

$500^2 + 1200^2 = z^2$

$z = 1300$

$1200(700) = 1300\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{8400}{13} ft/sec$

7. Given $\frac{dV}{dt} = -36\pi$ $V = \frac{4}{3}\pi r^3$

Find $\frac{dr}{dt}$ when $r = 5$

$\frac{dV}{dt} = \frac{4}{3}(3)\pi r^2 \frac{dr}{dt}$ (use chain rule)

$-36\pi = 4\pi(25)\frac{dr}{dt}$

$$\boxed{\frac{dr}{dt} = -\frac{9}{25} \text{ in/sec}}$$

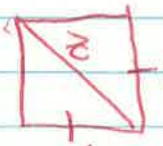
8. Given $dr/dt = -8$ in/hr $A = \pi r^2$

Find $\frac{dA}{dt}$ when $r = 3$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ (use chain rule)

$\frac{dA}{dt} = 2\pi(3)(-8)$

$\frac{dA}{dt} = -48\pi \text{ in}^2/\text{hr}$

9. 

$\frac{dz}{dt} = 4 \text{m/min}$ $A = x^2$

Find $\frac{dA}{dt}$ when $z = 14$

Find the relationship between the area of a square and its diagonal

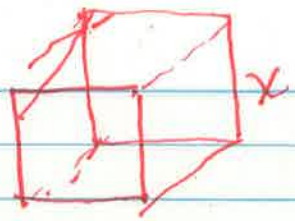
$x^2 + x^2 = z^2$ (Using Pythagorean)

$2x^2 = z^2$

$$\boxed{\frac{z^2}{2} = x^2}$$ therefore

$A = \frac{z^2}{2}$

$\frac{dA}{dt} = \frac{2z\,dz/dt}{2}$ $\frac{dA}{dt} = \frac{2(14)(4)}{2} = \boxed{56 \text{ m}^2/\text{min}}$

10. 

$\frac{dx}{dt} = 4 m/min$

Find ~~the volume~~ $\frac{dV}{dt}$ when $x = 7$

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3(7)^2(4)$

$\frac{dV}{dt} = 588 m^3/min$