AB Calculus Rate of Change Problems Name _____

1. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground.

   a) Write the equation of the distance of the ball after t seconds
   b) Find the average velocity of the ball from the time period of 3 seconds to 5 seconds.
   c) Find the velocity of the ball after 5 seconds
   d) At what time will the ball hit the ground?
   e) With what velocity will the ball hit the ground?

2. If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet after *t* seconds is given by $y = -16t^2 + 40t$

   a) Find the average velocity for the time period beginning when *t*=2 lasting
      i) .5 s ii) .1 s
      iii) .05 s iv) .01 s
   b) Find the instantaneous velocity when t=2.

3. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after *t* seconds is given by
   $$y = 58t - .83t^2$$
   a) Find the average velocity over the given time intervals:
      i) [1, 2] ii) [1, 1.5] iii) [1, 1.1]
      iv) [1, 1.01] v) [1, 1.001]
   b) Find the instantaneous velocity after 1s.
   c) Is the arrow rising or falling at this time?

4. The displacement in meters of a particle moving in a straight line is given by
   $$s = t^2 + t$$
   where *t* is measured in seconds.

   a) Find the average velocity over the following time periods:
      i) [0, 2] ii) [0, 1]
      iii) [0, .5] iv) [0, .1]
   b) Find the instantaneous velocity when *t* = 0.
   c) Draw a graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part a.
   d) Draw the tangent line whose slope is the instantaneous velocity in part b.

5. Temperature readings (in degrees C) where recorded every hour starting at 5:00 am on a day in April in Whitefish, Montana. This table shows some of the readings:

T	5	6	7	8	9	10	11	12	13
C	4	4	4.8	6.1	8.3	10	12.1	14.3	16

   Find the rate of average rate of change for each change in time:

   a) 8:00 am to 11:00 am
   b) 8:00 am to 10:00 am
   c) 8:00 am to 9:00 am
   d) Make a guess as to what the instantaneous rate of change was at 8:00am.

6. Suppose a ball is thrown into the air with a velocity of 40 ft/s.

   a) Write an equation in that models the relationship of distance and time.
   b) Find the velocity when *t*=2.

7. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters after *t* seconds is given by:

$$h = 58t - .83t^2$$

a) Find the velocity of the arrow at 1 s.
b) Find the velocity of the arrow when *t*=a.
c) When will the arrow hit the moon?
d) With what velocity will the arrow hit the moon?

8. The displacement in meters of a particle moving in a straight line is given by the equation of motion:
$s = 4t^3 + 6t + 2$ where *t* is measured in seconds. Find the velocity at times *t*=a, *t*=1, *t*=2, and *t*=3.

9. The population *P* (in thousands) of a city from 1980 to 1986 is given below:

Y	'80	'81	'82	'83	'84	'85	'86
P	105	110	117	126	137	150	164

a) Find the average rate of growth from:
i) '82 to '86 ii) '82 to '85
iii) '82 to '84 iv) '82 to '83
b) Estimate the instantaneous rate of growth in 1982.

10. The cost (in dollars) of producing *x* units of a certain commodity is:
$C(x) = 5000 + 10x + .05x^2$
Find the instantaneous rate of change of *C* with respect to *x* when *x* = 100. (This is called the marginal cost)

11. If a cylindrical tank holds 100,000 gallons of water, which takes 1 hour to drain from the bottom of the tank, then the volume of the water, *V*, remaining in the tank after *t* minutes is:

$$V(t) = 100{,}000\left(1 - \frac{t}{60}\right)^2,\ 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank after 20 min (multiply the problem out first).

12. Consider a particle moving along a linear path with distance, at time *t*, from 0 given by: $s(t) = 7t - t^2$

a) Find the particle's velocity function.
b) In the first 10 seconds, what is the time interval for which the point is moving to the left?

13. The position of a particle is given by the equation:
$s = f(t) = t^3 - 6t^2 + 9t$
where *t* is measured in seconds and *s* is measured in meters.

a) Find the velocity at time *t*.
b) What is the velocity at 2 s, at 4 s?
c) When is the particle at rest?
d) When is the particle moving in the positive direction?
e) Find the total distance traveled by the particle during the first 5 s.

14. A particle's motion is modeled by:
$f(t) = t^2 - 6t + 9$

a) Find the velocity at time t.
b) Find the velocity at 2 s.
c) When is the particle at rest?
d) When is the particle moving in a positive direction?
e) Find the total distance traveled in the first 4 s.

15. Find the average rate of change of the volume of a cube with respect to its edge length x as x changes from:

a) 5 to 6 b) 5 to 5.1 c) 5 to 5.01

d) Find the instantaneous rate of change when x = 5.