Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. E	2. B	3. C	4. D	5. C
6. E	7. E	8. C	9. B	10. C
11. A	12. A	13. A	14. A	15. A
16. D	17. C	18. D	19. A	20. C
21. A	22. C	23. B	24. D	25. D
26. B	27. C	28. E	29. B	30. D
31. D	32. C	33. C	34. E	35. B
36. A	37. C	38. B	39. D	40. E
41. D	42. E	43. A	44. B	45. D

MULTIPLE-CHOICE QUESTIONS

***Note*: Asterisks (*) indicate BC questions and solutions.**

1. **ANSWER: (E)** $f'(x) = 2x(-\sin x) + \cos x\,(2) = -2x\sin x + 2\cos x$ (*Calculus, Early Transcendentals* 7th ed. pages 191–194 / 8th ed. pages 190–193).

2. **ANSWER: (B)** $f'(x)$ increasing $\Rightarrow f''(x) > 0 \Rightarrow f(x)$ is concave up.
$f'(x)$ decreasing $\Rightarrow f''(x) < 0 \Rightarrow f(x)$ is concave down.
(B) is concave up for $x < 2$ and concave down for $x > 2$ (*Calculus, Early Transcendentals* 7th ed. pages 310–316 / 8th ed. pages 315–321).

3. **ANSWER: (C)** If the interval from 0 to 1 is partitioned into n subintervals, then each one has width $\Delta x = \frac{1}{n}$ and their x-coordinates are $\frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{k}{n}, \ldots, \frac{n}{n}$. Thus $x_k = \frac{k}{n}$. Recall that a definite integral is defined as the limit of a Riemann sum, $\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x$. In this problem, $f(x) = \sqrt{x}$. Therefore,
$\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\cdot\frac{1}{n} = \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{x_k}\,\Delta x = \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}$ (*Calculus, Early Transcendentals* 7th ed. pages 371–380 / 8th ed. pages 378–386).

*4. **ANSWER: (D)** Using Euler's method, $y_{n+1} \approx y_n + \left.\frac{dy}{dx}\right|_{(x_n, y_n)} \cdot \Delta x$

$x_0 = 2$ $\quad y_0 = 5$ $\quad \left.\frac{dy}{dx}\right|_{(2,\,5)} = 5 - 4 + 3 = 4$

$x_1 = 2.5$ $\quad y_1 = 5 + 4(0.5) = 7$ $\quad \left.\frac{dy}{dx}\right|_{(2.5,\,7)} = 7 - 5 + 3 = 5$

$x_2 = 3$ $\quad y_2 = 7 + 5(0.5) = 9.5$ $\quad$ Therefore, $f(3) \approx 9.5$

(*Calculus, Early Transcendentals* 7th ed. pages 589–591 / 8th ed. pages 595–597).

5. **ANSWER: (C)** $y(5) = 8\sqrt{3 \cdot 5 + 1} = 32$

$$y' = \frac{8 \cdot 3}{2\sqrt{3x+1}} = \frac{12}{\sqrt{3x+1}} \Rightarrow y'(5) = \frac{12}{\sqrt{3 \cdot 5 + 1}} = 3$$

The equation of the tangent line is $y - 32 = 3(x - 5)$, or $y = 3x + 17$ (*Calculus, Early Transcendentals* 7th ed. pages 198–205 / 8th ed. pages 197–204).

*6. **ANSWER: (E)** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{16}{2\sqrt{t}}}{\frac{10}{1+t}} = \frac{16(1+t)}{20\sqrt{t}}$. Therefore,

$$\left.\frac{dy}{dx}\right|_{t=4} = \frac{80}{40} = 2$$

(*Calculus, Early Transcendentals* 7th ed. pages 636–647 / 8th ed. pages 640–651).

7. **ANSWER: (E)** $\int_0^1 \frac{3}{x}\,dx = \lim_{b \to 0} \int_b^1 \frac{3}{x}\,dx = \lim_{b \to 0} 3\ln|x|\Big|_b^1 =$ $3\ln|1| - \lim_{b \to 0} 3\ln|b| = 0 - (-\infty) = \infty$ (*Calculus, Early Transcendentals* 7th ed. pages 519–526 / 8th ed. pages 527–534).

8. **ANSWER: (C)** Since $f(x)$ is strictly increasing, left end points produce inscribed rectangles and an underapproximation. Right end points produce circumscribed rectangles and an overapproximation. Since $f(x)$ is concave down, a trapezoidal approximation consists of line segments which are below $f(x)$, producing an underapproximation. So I and II are false and III is true, making (C) correct (*Calculus, Early Transcendentals* 7th ed. pages 371–380, 508–511 / 8th ed. pages 378–386, 516–519).

9. **ANSWER: (B)** Separating variables,

$$\int \frac{dy}{y} = \int \left(1 + \frac{1}{x^2}\right) dx \Rightarrow \ln|y| = x - \frac{1}{x} + C_1 \Rightarrow y = e^{x - \frac{1}{x} + C_1} \Rightarrow y = Ce^{x - \frac{1}{x}}$$

(*Calculus, Early Transcendentals* 7th ed. pages 594–597 / 8th ed. pages 599–602).

10. **ANSWER: (C)** Differentiating both sides implicitly,
$2x \cdot 2y\dfrac{dy}{dx} + 2y^2 = 6x - 3y^2\dfrac{dy}{dx}$.

At point (1, 1), this equation is $4\dfrac{dy}{dx} + 2 = 6 - 3\dfrac{dy}{dx}$. Therefore,
$\left.\dfrac{dy}{dx}\right|_{(1,1)} = \dfrac{6-2}{4+3} = \dfrac{4}{7}$ (*Calculus, Early Transcendentals*
7th ed. pages 209–214 / 8th ed. pages 208–214).

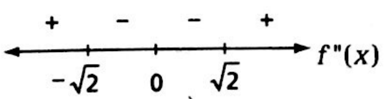
11. **ANSWER: (A)** $f(x) = \int 12x^2 \sin\left(2x^3 - 16\right) dx$. Let
$u = 2x^3 - 16 \Rightarrow du = 6x^2 dx$.
$f(x) = \int 2 \sin u \, du = -2\cos u + C = -2\cos\left(2x^3 - 16\right) + C$.
$5 = -2\cos(2 \cdot 2^3 - 16) + C \Rightarrow 5 = -2\cos(0) + C = -2 + C \Rightarrow C = 7$.
$f(x) = -2\cos\left(2x^3 - 16\right) + 7$ (*Calculus, Early Transcendentals*
7th ed. pages 344–348 / 8th ed. pages 350–355).

*12. **ANSWER: (A)** The Taylor expansion for a function about $x = a$ is defined as $\sum_{k=0}^{\infty} \dfrac{f^{(k)}(a)(x-a)^k}{k!}$. Therefore, $\dfrac{f''(3)(x-3)^2}{2!} = -\dfrac{7(x-3)^2}{3}$.
Solving, $f''(3) = -\dfrac{7}{3} \cdot 2! = -\dfrac{14}{3}$.

Equivalently, differentiating the given polynomial twice and substituting $x = 3$ produces $f''(3) = -14/3$ (*Calculus, Early Transcendentals* 7th ed. pages 753–764 / 7th ed. SV pages 775–786 / 8th ed. pages 759–770).

13. **ANSWER: (A)** $f'(x) = 6x^5 - 20x^3 \Rightarrow f''(x) = 30x^4 - 60x^2 = 30x^2\left(x^2 - 2\right)$

$30x^2\left(x^2 - 2\right) = 0 \Rightarrow x = 0, \ \pm\sqrt{2}$



The sign of $f''(x)$ changes at $x = \pm\sqrt{2}$ only, so these are the locations of the inflection points (*Calculus, Early Transcendentals*
7th ed. pages 292–294 / 8th ed. pages 292–295).

*14. **ANSWER: (A)** $\dfrac{\cos(0) - e^0}{\ln(1+0)} = \dfrac{1-1}{0} = \dfrac{0}{0}$. This is a quotient indeterminate form, so L'Hôpital's rule applies.
$\lim_{x \to 0} \dfrac{\cos x - e^x}{\ln(1+x)} = \lim_{x \to 0} \dfrac{-\sin x - e^x}{\dfrac{1}{1+x}} = \dfrac{-0-1}{\dfrac{1}{1+0}} = -1$
(*Calculus, Early Transcendentals* 7th ed. pages 301–307 /
8th ed. pages 304–311).

*15. **Answer: (A)** I: The series is geometric with $r = 3/4$, so it converges.

II: $\lim_{k\to\infty} \frac{k^2}{(2k+1)^2} = \frac{1}{4} \neq 0$, so the series diverges by the nth-Term Test.

III: $|\sec k| \geq 1$, so $\frac{|\sec k|}{k} \geq \frac{1}{k}$. Since $\sum_{k=1}^{\infty} \frac{1}{k}$ is a divergent p-series (harmonic series, $p = 1$), the series $\sum_{k=1}^{\infty} \frac{|\sec k|}{k}$ diverges by the Direct Comparison Test. Therefore, (A) is correct (*Calculus, Early Transcendentals* 7th ed. pages 706–707, 722–723 / 7th ed. SV pages 728–729, 744–745 / 8th ed. pages 710–711, 727–728).

16. **Answer: (D)** Separating variables,

$$\int \frac{dy}{y-2} = \int k\,dt \Rightarrow \ln|y-2| = kt + C_1 \Rightarrow y = e^{kt+C_1} + 2 \Rightarrow y = Ce^{kt} + 2$$

(*Calculus, Early Transcendentals* 7th ed. pages 594–597 / 8th ed. pages 599–602).

17. **Answer: (C)** By The Fundamental Theorem Part 1, the domain is the largest continuous interval of $f(t)$ containing the lower limit of the integral. Since the upper limit is a function of x, solve the inequality $-1 < 2x - 1 < 5$. The solution is $0 < x < 3$ (*Calculus, Early Transcendentals* 7th ed. pages 386–394 / 8th ed. pages 382–399).

18. **Answer: (D)** $F'(x) = f(2x-1)\cdot 2 \Rightarrow F'(2) = 2f(2\cdot 2 - 1) = 2f(3) = 2\cdot 2 = 4$ (*Calculus, Early Transcendentals* 7th ed. pages 386–394 / 8th ed. pages 392–399).

19. **Answer: (A)** $f'(x) = \frac{3}{\sqrt{1-x^2}} \Rightarrow f'(0) = \frac{3}{1} = 3 \Rightarrow -\frac{1}{f'(0)} = -\frac{1}{3}$ (*Calculus, Early Transcendentals* 7th ed. pages 67–69, 201 / 8th ed. pages 63–66, 200).

*20. **Answer: (C)** $\frac{dx}{dt} = \sec^2 t$ and $\frac{dy}{dt} = \frac{1}{2}e^{\frac{1}{2}t}$

Length $= \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{\sec^4 t + \frac{1}{4}e^t}\,dt$ (*Calculus, Early Transcendentals* 7th ed. pages 636–647 / 8th ed. pages 640–651).

*21. **Answer: (A)** The Maclaurin series for e^x is $\sum_{k=0}^{\infty} \frac{x^k}{k!}$. So the series for e^{x^2} is $\sum_{k=0}^{\infty} \frac{(x^2)^k}{k!} = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$.

$$\text{Area} = \int_0^1 e^{x^2}dx = \int_0^1\left(\sum_{k=0}^{\infty}\frac{x^{2k}}{k!}\right)dx \approx \int_0^1\left(\frac{x^0}{0!}+\frac{x^2}{1!}+\frac{x^4}{2!}+\frac{x^6}{3!}\right)dx$$

$$= \frac{x^1}{1\cdot 0!}+\frac{x^3}{3\cdot 1!}+\frac{x^5}{5\cdot 2!}+\frac{x^7}{7\cdot 3!}\Bigg|_0^1 = 1+\frac{1}{3}+\frac{1}{10}+\frac{1}{42}$$

(*Calculus, Early Transcendentals* 7th ed. pages 753–764 / 7th ed. SV pages 775–786 / 8th ed. pages 759–770).

22. **ANSWER: (C)** The slopes are positive in quadrants I and II and negative in quadrants III and IV. This indicates no change in sign on opposite sides of the y-axis, thus x has an even power. There is a change in sign on opposite sides of the x-axis, thus y has an odd power. Therefore, (C) is correct (*Calculus, Early Transcendentals* 7th ed. pages 585–589 / 8th ed. pages 591–594).

•23. **ANSWER: (B)** The answer can be determined in two ways: 1. Find the series for $f(x) = x\sin x$ and differentiate. 2. Differentiate $f(x) = x\sin x$ and find its series.

$$1:\quad f(x) = x\sin x \approx x\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}\right)$$

$$= x^2-\frac{x^4}{3!}+\frac{x^6}{5!}-\frac{x^8}{7!}.\quad f'(x)\approx 2x-\frac{4x^3}{3!}+\frac{6x^5}{5!}-\frac{8x^7}{7!}$$

$$2:\quad f'(x) = x\cos x+\sin x \approx x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\right)+x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$$

$$= x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$$

$$= 2x-\frac{4x^3}{3!}+\frac{6x^5}{5!}-\frac{8x^7}{7!}$$

(*Calculus, Early Transcendentals* 7th ed. pages 753–764 / 7th ed. SV pages 775–786 / 8th ed. pages 759–770).

•24. **ANSWER: (D)** $r = \dfrac{1}{\cos\left(\frac{1}{2}\theta\right)} = \sec\left(\frac{1}{2}\theta\right)$

$$\text{Polar area} = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}}\sec^2\frac{1}{2}\theta\,d\theta$$

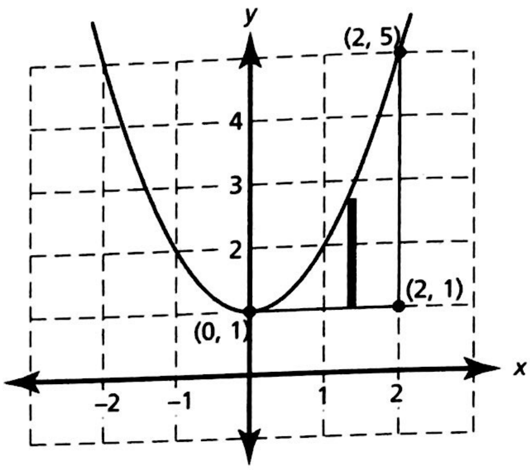
$$= 2\cdot\frac{1}{2}\tan\frac{1}{2}\theta\Big|_0^{\frac{\pi}{2}}$$

$$= \tan\frac{\pi}{4}-\tan 0$$

$$= 1-0 = 1$$

(*Calculus, Early Transcendentals* 7th ed. pages 665–667 / 7th ed. SV pages 684–686 / 8th ed. pages 669–671).

25. **ANSWER: (D)** By the Washer Method,
Volume $= \pi \int_0^2 \left[\left(1+x^2\right)^2 - 1^2\right] dx$
(*Calculus, Early Transcendentals* 7th ed. pages 434–436) /
8th ed. pages 440–444).



*26. **ANSWER: (B)**
$\int \frac{2x-3}{x^2+9x+18} dx = \int \frac{2x-3}{(x+6)(x+3)} dx.$
Integrate by partial fractions.
$\frac{2x-3}{(x+6)(x+3)} = \frac{A}{x+6} + \frac{B}{x+3}$
$\Rightarrow 2x-3 = A(x+3) + B(x+6)$

Let $x=-6$: $2(-6)-3 = A(-6+3) \Rightarrow -15 = 3A \Rightarrow A = 5$

Let $x=-3$: $2(-3)-3 = B(-3+6) \Rightarrow -9 = 3B \Rightarrow B = -3$

Therefore,

$$\int \frac{2x-3}{(x+6)(x+3)} dx = \int \left(\frac{5}{x+6} - \frac{3}{x+3}\right) dx$$
$$= 5 \ln|x+6| - 3 \ln|x+3| + C$$
$$= \ln\left|\frac{(x+6)^5}{(x+3)^3}\right| + C$$

(*Calculus, Early Transcendentals* 7th ed. pages 484–492 /
8th ed. pages 493–500).

*27. **ANSWER: (C)** To get the velocity vector, integrate the coordinates of the acceleration vector.
$\int -\pi \sin \pi t \, dt = \cos \pi t + C_1$. $\cos(\pi \cdot 0) + C_1 = 1 \Rightarrow 1 + C_1 = 1 \Rightarrow C_1 = 0.$

$\int (2t+1)\, dt = t^2 + t + C_2$. $0^2 + 0 + C_2 = 0 \Rightarrow C_2 = 0$. The velocity vector is $\left(\cos \pi t,\ t^2 + t\right)$. Therefore the speed of the particle when $t = 2$ is $\sqrt{\cos^2(\pi \cdot 2) + \left(2^2+2\right)^2} = \sqrt{1^2 + 6^2} = \sqrt{37}$ (*Calculus, Early Transcendentals* 7th ed. pages 636–647 / 8th ed. pages 640–651).

*28. **ANSWER: (E)** This is a variation on a *p*-series, $\sum_{k=1}^{\infty} \frac{1}{k^p}$, so the Limit Comparison Test should be used. If $\lim_{k\to\infty} \frac{u_k}{v_k}$ is finite and positive, then the original series and the comparison series will both converge or both diverge. A *p*-series converges if $p > 1$. Compare to $\sum_{k=1}^{\infty} \frac{1}{k^{2a-5}}$, because the difference in degree (denominator minus numerator) of the original series is $2a - 3 - 2 = 2a - 5$.

$$\lim_{k\to\infty}\frac{\frac{k^2}{k^{2a-3}+4}}{\frac{1}{k^{2a-5}}}=\lim_{k\to\infty}\frac{k^2\cdot k^{2a-5}}{k^{2a-3}+4}=\lim_{k\to\infty}\frac{k^{2a-3}}{k^{2a-3}+4}=1,$$ which is finite and positive. $\sum_{k=1}^{\infty}\frac{1}{k^{2a-5}}$ converges if $2a-5>1 \Rightarrow 2a>6 \Rightarrow a>3$ (*Calculus, Early Transcendentals* 7th ed. pages 708, 714–717, 724/7th ed. SV pages 730, 736–739, 746 / 8th ed. pages 712, 719–722, 729).

29. **ANSWER: (B)** Using the Trapezoid Rule,

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n)$$
$$=\frac{10-4}{2\cdot 3}(24+2\cdot 37+2\cdot 47+58)=1(250)=250$$

(*Calculus, Early Transcendentals* 7th ed. pages 508–511 / 8th ed. pages 516–519).

30. **ANSWER: (D)** $\lim_{x\to -2} f(x)=+\infty$, which is nonexistent; the graph is closed at $x = 1$, thus $f(1)$ exists; $f(x)$ is not continuous at $x = 1$, and therefore cannot be differentiable at $x = 1$; $\lim_{x\to\infty} f(x)=0$, indicated by the horizontal asymptote $y = 0$. Thus (A), (B), (C), and (E) are all false. $\lim_{x\to 1^+} f(x)$ is a finite value, even though it is not the same value as $f(1)$. Therefore, (D) is the true statement (*Calculus, Early Transcendentals* 7th ed. pages 118–127 / 8th ed. pages 114–123).

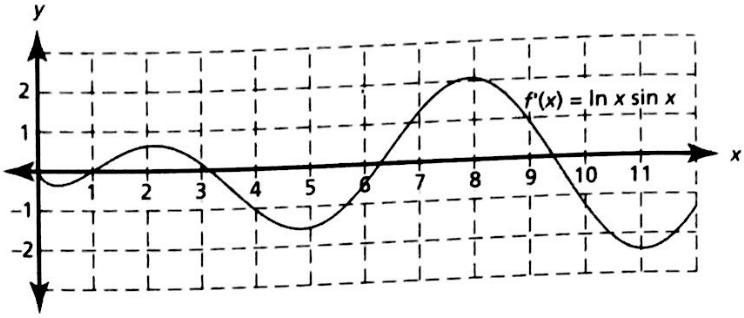
31. **ANSWER: (D)** $f(3)=0+\int_{-3}^{3} f'(x)\,dx$ and represents the accumulated area under the curve from $x=-3$ to $x=3$. The net signed areas of the triangles are $\frac{1}{2}\cdot 3\cdot 2+\frac{1}{2}\cdot 2\cdot 2-\frac{1}{2}.1\cdot 1=3+2-\frac{1}{2}=4.5$ (*Calculus, Early Transcendentals* 7th ed. pages 397–413 / 8th ed. pages 402–418).

*32. **ANSWER: (C)** $dy/dx=\dfrac{dy/dt}{dx/dt}\Rightarrow dx/dt=\dfrac{dy/dt}{dy/dx}$. For the given function,

$\frac{dy}{dx}=x\cos x+\sin x \Rightarrow \left.\frac{dy}{dx}\right|_{x=3}=3\cos 3+\sin 3$. Therefore,

$\left.\frac{dx}{dt}\right|_{x=3}=\frac{-2}{3\cos 3+\sin 3}\approx 0.707$ (*Calculus, Early Transcendentals* 7th ed. pages 636–647 / 8th ed. pages 640–651).

33. **ANSWER: (C)** $1.6 \le x \le 11.6$. In looking at the graph of $f'(x)$ on the given window, there are four turning points. This represents four points at which $f''(x)$ (or the slope of $f'(x)$) is equal to 0 and changes sign from either positive to negative or negative to positive. So these represent four changes in concavity, hence four inflection points (*Calculus, Early Transcendentals* 7th ed. pages 290–297 / 8th ed. pages 293–300).



34. **ANSWER: (E)** The average value of a function in an interval is the value of the definite integral divided by its length, that is, $f_{ave} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$. A look at the graphs of $y = \cos x$, $y = \cos 2x$, and $y = \sin x$ reveals that the areas under the curves can be easily compared without computation. (A) and (C) have positive areas, (B) and (D) are zero, and (E) is negative. Therefore, (E) is the only choice to have a negative average value, so it is the smallest. Alternatively, calculate the five average values on the graphing calculator and see that (E) is the smallest (*Calculus, Early Transcendentals* 7th ed. pages 451–453 / 8th ed. pages 461–462).

35. **ANSWER: (B)** Use the graphing calculator to graph $v(t)$. Use the derivative feature to graph $v'(t)$. Then $v'(t) = a(t) = 0$ at $t = 2.35619$, $v(2.35619) = -0.670$ (*Calculus, Early Transcendentals* 7th ed. pages 148, 161 / 8th ed. pages 145–146, 160).

*36. **ANSWER: (A)** Series I and II are essentially p-series, so the convergence of the series of absolute values can be obtained by using the Limit Comparison Test. Recall that $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $0 < p \le 1$. Compare I to $\sum_{k=1}^{\infty} \frac{1}{k}$, which is divergent. $\lim_{k\to\infty} \left| \dfrac{(-1)^k \frac{k^2}{k^3+1}}{\frac{1}{k}} \right| = \lim_{k\to\infty} \left| \dfrac{k^3}{k^3+1} \right| = 1$, which is finite and positive, thus I does not converge absolutely. But the sequence of positive terms decreases to a limit of zero, so as an alternating series, I converges by the Alternating Series Test. Therefore, I is conditionally convergent. Compare II to $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Using the same limit procedure, II converges absolutely. The sequence in III has limit 1, so it is divergent. In summary, I is the only series that converges conditionally (*Calculus, Early Transcendentals* 7th ed. pages 706, 714–717, 724 / 7th ed. SV pages 730, 736–739, 746 / 8th ed. pages 710, 719–722, 729).

37. **ANSWER: (C)** $\tan\theta = \frac{h}{8} \Rightarrow h = 8\tan\theta \Rightarrow \frac{dh}{dt} = 8\sec^2\theta\frac{d\theta}{dt}$. When $h = 13,\ 8^2 + 13^2 = z^2 \Rightarrow z = \sqrt{64+169} = \sqrt{233}$.
$\left.\frac{dh}{dt}\right|_{h=13} = 8\cdot\frac{233}{64}(0.03) = 0.874$ cm/sec (*Calculus, Early Transcendentals* 7th ed. pages 244–248 / 8th ed. pages 245–248).

38. **ANSWER: (B)** $f''(x) = e^x + 1$, which is positive everywhere. Therefore, $f(x)$ is concave up everywhere, so any tangent line to $f(x)$ will be below the curve except at the point of tangency. Thus, $H(a + 0.1) < f(a + 0.1)$ for all values of a. [Note: If, for example, $f''(x) = e^x - 1$, there would be a sign change in $f''(x)$, hence an inflection point on the graph of $f(x)$. In that case, (C) would be the correct answer.] (*Calculus, Early Transcendentals* 7th ed. pages 290–297, 310–316, 325–331 / 8th ed. pages 293–300, 315–321, 330–336).

*39. **ANSWER: (D)** Using the Ratio Test for Absolute Convergence,

$$\lim_{k\to\infty}\left|\frac{\frac{(2x)^{k+1}}{k+2}}{\frac{(2x)^k}{k+1}}\right| = \lim_{k\to\infty}\left|\frac{(2x)^{k+1}\cdot(k+1)}{(k+2)\cdot(2x)^k}\right| < 1$$

$$\Rightarrow \lim_{k\to\infty}|2x| < 1 \Rightarrow |2x| < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

End points must be checked separately.

$x = -\frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}$ converges by the Alternating Series Test, since the series of positive terms is decreasing and $\lim_{k\to\infty}\frac{1}{k+1} = 0$.

$x = \frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{1}{k+1}$ diverges, since it is a p-series (harmonic series, $p = 1$). Therefore, the interval of convergence is $-\frac{1}{2} \le x < \frac{1}{2}$ (*Calculus, Early Transcendentals* 7th ed. pages 708, 714–717, 727–737 / 7th ed. SV pages 730, 736–739, 749–759 / 8th ed. pages 712, 719–722, 732–742).

40. **ANSWER: (E)** The functions intersect at $x = -6$, 0, and 4 and enclose two regions.

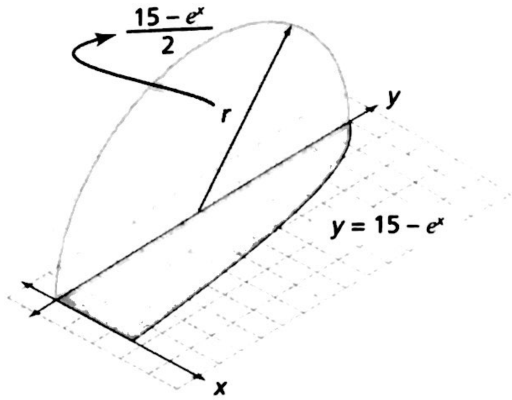
Area = $\int_{-6}^{0}[f(x) - g(x)]\,dx + \int_{0}^{4}[g(x) - f(x)]\,dx = 31.5 + 10.667 = 42.167$. Alternatively, Area = $\int_{-6}^{4}|f(x) - g(x)|\,dx = 42.167$ (*Calculus, Early Transcendentals* 7th ed. pages 422–426 / 8th ed. pages 428–434).

41. **ANSWER: (D)** The total sales figure is represented by $\int_0^{30}(0.32x^2 - 0.01x^3)\,dx = 855$ (*Calculus, Early Transcendentals* 7th ed. pages 397–403 / 8th ed. pages 402–408).

42. **ANSWER: (E)** By the product and chain rules, $g'(x) = x^2 \cdot 3f'(3x) + 2x \cdot f(3x)$. Therefore, $g'(-1) = (-1)^2 \cdot 3f'(-3) - 2f(-3) = 3 \cdot 7 - 2(-2) = 25$ (*Calculus, Early Transcendentals* 7th ed. pages 185, 198–205 / 8th ed. pages 184, 197–204).

43. **ANSWER: (A)** $15 - e^x = 0 \Rightarrow x = 2.70805$. The radius of each cross section is $\frac{15 - e^x}{2}$, so the area of each cross section is $\frac{1}{2}\pi\left(\frac{15 - e^x}{2}\right)^2 = \frac{\pi}{8}\left(15 - e^x\right)^2$.

Therefore, $V = \int_0^{2.70805} \frac{\pi}{8}\left(15 - e^x\right)^2 \, dx = 118.325$ (*Calculus, Early Transcendentals* 7th ed. pages 436–438 / 8th ed. pages 444–446).



*44. **ANSWER: (B)** This is the indeterminate form ∞^0.
Let $y = [f(x)]^{\frac{1}{x}}$. Then $\ln y = \frac{1}{x}\ln[f(x)] = \frac{\ln[f(x)]}{x}$.
Thus $\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln[f(x)]}{x}$. This is the indeterminate form $\frac{\infty}{\infty}$, so use L'Hôpital's rule. $\lim_{x \to \infty} \frac{\ln[f(x)]}{x} = \lim_{x \to \infty} \frac{\frac{f'(x)}{f(x)}}{1} = \frac{\frac{3}{\infty}}{1} = 0$. Therefore, $\lim_{x \to \infty} \ln y = 0 \Rightarrow \lim_{x \to \infty} y = e^0 = 1$ (*Calculus, Early Transcendentals* 7th ed. pages 301–307 / 8th ed. pages 304–311).

45. **ANSWER: (D)** The Mean Value Theorem guarantees at least one value c in the interval $0.5 < x < 3.5$ such that $f'(c) = \frac{f(3.5) - f(0.5)}{3.5 - 0.5}$. For the given function, $f'(x) = \frac{1}{x} - 1$. Therefore,
$f'(c) = \frac{1}{c} - 1 = \frac{0.75276 - 1.80685}{3.5 - 0.5} = -\frac{1.05409}{3} = -0.35136$. So
$\frac{1}{c} - 1 = -0.35136 \Rightarrow c = 1.542$ (*Calculus, Early Transcendentals* 7th ed. pages 285–286 / 8th ed. pages 288–289).