KEY

Determine if the SEQUENCES converge or diverge. If it converges, find its limit.

1. $a_n = \frac{n^2-7n+3}{1+10n-4n^2}$ C, $-\frac{1}{4}$

2. $a_n = \frac{e^{5n}}{3-e^{2n}}$ D

3. $a_n = \frac{\ln(n+2)}{\ln(1+4n)}$ C, 1

Determine if the SERIES converge or diverge.

4. $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ D int	5. $\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^2+1}}$ D nth
6. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt[4]{n}}$ C pseries	7. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$ D nth
8. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ C geo	9. $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$ C int

	Series	Converges/ Diverges	Which Test?
10.	$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$	D	p-series
11.	$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$	C	integral
12.	$\sum_{n=1}^{\infty}\frac{2^{n+1}}{5^{n-1}}$	C	geometric
13.	$\sum_{n=1}^{\infty}\frac{n+2}{n+1}$	D	nth term

Quiz Review

1) $a_n = \dfrac{n^2-7n+3}{1+10n-4n^2}$ $\quad \lim\limits_{n\to\infty} a_n = -\dfrac{1}{4}$ converges

2) $a_n = \dfrac{e^{5n}}{3-e^{2n}}$ $\quad \lim\limits_{n\to\infty} \dfrac{e^{5n}}{3-e^{2n}}$ L'H: $\dfrac{5e^{5n}}{-2e^{2n}} = \dfrac{-5}{2}e^{3n}$

$= -\infty$ $\therefore$ diverges

3) $a_n = \dfrac{\ln(n+2)}{\ln(1+4n)}$ $\quad \lim\limits_{n\to\infty} \dfrac{\ln(n+2)}{\ln(1+4n)}$ L'H: $\dfrac{\frac{1}{n+2}}{\frac{1}{1+4n}\cdot 4}$

$= \dfrac{1+4n}{4(n+2)} = \dfrac{1+4n}{4n+8}$ L'H: $\dfrac{4}{4} = 1$ converges

4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ $\quad \lim\limits_{n\to\infty} \dfrac{1}{2n-1} = 0$

$\displaystyle\int_1^{\infty} \frac{1}{2x-1}dx = \lim_{b\to\infty} \int_1^b \frac{1}{2x-1}dx$ $\quad u = 2x-1$, $du = 2dx$, $\frac{1}{2}du = dx$

$= \displaystyle\frac{1}{2}\int_1^b \frac{1}{u}du = \frac{1}{2}\ln|2x-1| \Big|_1^b$

$= \dfrac{1}{2}\left(\ln|2b-1| - \cancel{\ln 1}^{\,0}\right) = \infty$ $\therefore$ diverges

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5) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ $\quad \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$

$\therefore$ diverges

6) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}} = \frac{1}{n^{5/4}}$ $\quad$ p-series $p>1$ $\therefore$ converges

7) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ $\quad \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$ $\quad 1^{\infty}$

$\ln y = \lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right)$

$\lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$ $\quad$ L'H: $\frac{\frac{1}{1+\frac{1}{n}} \cdot \cancel{-1n^{-2}}}{\cancel{-1n^{-2}}}$

$\ln y = 1$

$y = e \neq 0$ diverges

8) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ $\quad r = \frac{2}{3}$ $\quad \left|\frac{2}{3}\right| < 1$ converges

9) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ $\quad \lim_{n\to\infty} \frac{1}{n(\ln n)^3} = 0$

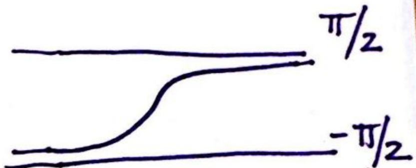
$\int \frac{1}{x(\ln x)^3}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$\int u^{-3}\,du$ $\quad \frac{-1}{2u^2} = \left.\frac{-1}{2(\ln x)^2}\right|_2^b = \frac{-1}{2(\ln b)^2} - \left(\frac{-1}{2(\ln 2)^2}\right)$

$\therefore$ converges

10) $\frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$ $p<1$ $\therefore$ diverges p-series

11) $\frac{1}{n^2+1}$ $\lim_{n \to \infty} = 0$



$$\int \frac{1}{x^2+1}\,dx = \arctan x \Big|_1^{\infty} = \arctan \infty - \arctan 1$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{2}$$

$\therefore$ converges
integral test

12) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^{n-1}}$ $\frac{2^n \cdot 2^1}{5^n \cdot 5^{-1}} = 10\left(\frac{2}{5}\right)^n$

geometric $r = \frac{2}{5}$ $\left|\frac{2}{5}\right| < 1$

$\therefore$ converges

13) $\frac{n+2}{n+1}$ $\lim_{n \to \infty} \frac{n+2}{n+1}$ L'H: $\frac{1}{1} = 1 \neq 0$

$\therefore$ diverges
n^{th} term test