Quiz 2 Review (Theorems):

1) $f(x) = x^4 - 2x^2 + 3 \quad [-2,2]$

✓ $f(x)$ is cont on $[-2,2]$
✓ $f(x)$ is diff on $(-2,2)$
✓ $f(-2) = f(2) = 11$

$f(-2) = (-2)^4 - 2(-2)^2 + 3$
$16 - 8 + 3$
11

$f(2) = 2^4 - 2(2)^2 + 3$
$f(2) = 16 - 8 + 3$
11

$f'(x) = 4x^3 - 4x = 0$
$4x(x^2 - 1) = 0$
$x = 0, 1, -1$

$\boxed{c = -1, 0, 1}$

2) $f(x) = x^2 - 3x + 2 \quad [-2,3]$

✓ $f(x)$ is cont on $[-2,3]$
✓ $f(x)$ is diff on $(-2,3)$

$f(3) = 9 - 9 + 2 = 2$
$f(-2) = 4 + 6 + 2 = 12$

$$f'(x) = \frac{f(3) - f(-2)}{3 - (-2)}$$

$$2x - 3 = \frac{2 - 12}{5}$$

$2x - 3 = -2$
$2x = 1$
$x = \frac{1}{2}$

$\boxed{c = \frac{1}{2}}$

3) $f(x) = x^3 - 2x^2 + x \quad [1, 2]$

$f'(x) = 3x^2 - 4x + 1$

$f'(x) = (3x - 1)(x - 1)$

$x = \frac{1}{3}, \quad x = 1$

x	y
-1	-4
1/3	4/27
1	0
2	2

Abs max at (2, 2)

Abs min at (-1, -4)

4) $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2 - 1) = 0$

$x = 0, 1, -1$

crit pts: $x = -1, 0, 1$

Inc: $(-1, 0) \cup (0, 1)$ b/c $f'(x)$ is pos

Dec: $(-\infty, -1) \cup (1, \infty)$ b/c $f'(x)$ is neg

5) $f'(x) = x^2 - \frac{2}{x}$

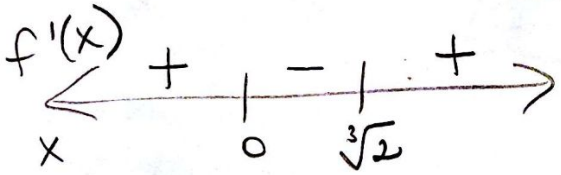
$f'(x) = \frac{x^3 - 2}{x}$

$x^3 - 2 = 0$

$x^3 = 2$

$x = \sqrt[3]{2}$

$x = 0$



Dec: $(0, \sqrt[3]{2})$

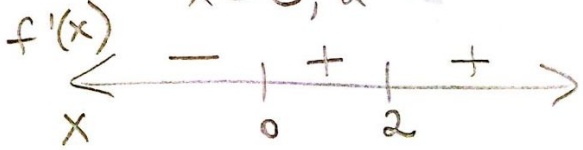
6) $f(x) = 3x^4 - 16x^3 + 24x^2 + 48$

$f'(x) = 12x^3 - 48x^2 + 48x$

$f'(x) = 12x(x^2 - 4x + 4)$

$f'(x) = 12x(x-2)(x-2)$

$x = 0, 2$



Dec: $(-\infty, 0)$ b/c $f'(x)$ is neg

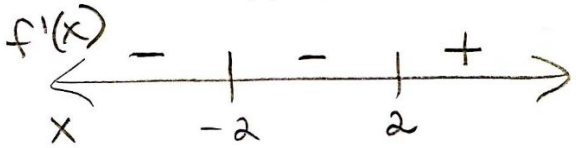
7) $f'(x) = \dfrac{|4-x^2|}{x-2}$

critical pts:

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$



Dec: $(-\infty, -2) \cup (-2, 2)$

8) $f(x) = 5x^3 - 7x^2 - 8x$

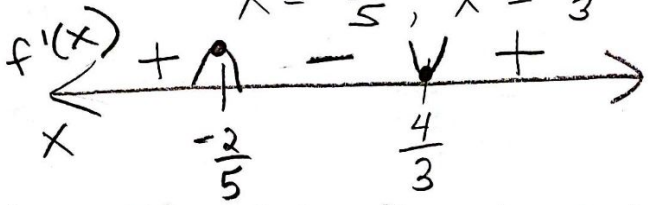
$f'(x) = 15x^2 - 14x - 8 = 0$

$15x^2 + 6x - 20x - 8 = 0$

$3x(5x+2) - 4(5x+2) = 0$

$(5x+2)(3x-4) = 0$

$x = \frac{-2}{5}, \quad x = \frac{4}{3}$



Inc: $(-\infty, -2/5) \cup (4/3, \infty)$ b/c $f'(x)$ is pos

Dec: $(-2/5, 4/3)$ b/c $f'(x)$ is neg

Rel max: $(-2/5, 44/25)$

Rel min: $(4/3, -304/27)$

9) $y = \frac{x^2+1}{x^2-4}$

$$y' = \frac{(x^2-4)(2x) - [(x^2+1)(2x)]}{(x^2-4)^2}$$

($2x^3 + 2x$)

$$y' = \frac{2x^3 - 8x - 2x^3 - 2x}{(x^2-4)^2}$$

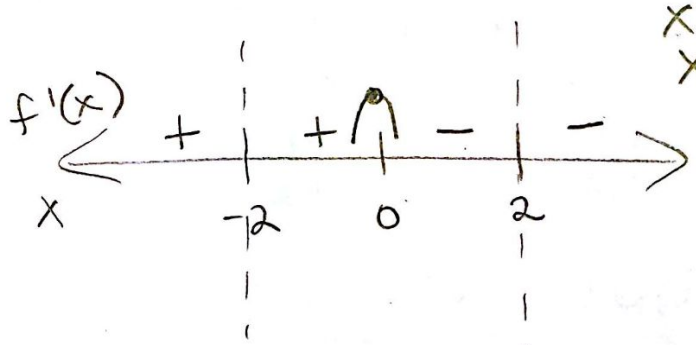
$$y' = \frac{-10x}{(x^2-4)^2}$$

crit pts:

$x = 0$

$x = 2$

$x = -2$



Inc: $(-\infty, -2) \cup (-2, 0)$ b/c $f'(x)$ is pos

Dec: $(0, 2) \cup (2, \infty)$ b/c $f'(x)$ is neg

Rel max: $(0, -1/4)$