— Stations Review:

$\boxed{1}$ a) $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$

$\int 3y^2\,dy = \int x^2+2\,dx$

$y^3 = \frac{x^3}{3} + 2x + C$

$y = \sqrt[3]{\frac{x^3}{3} + 2x + C}$

b) $\frac{dr}{ds} = 0.05s$

$\int dr = \int 0.05s\,ds$

$r = .025s^2 + C$

c) $xy' = y$

$x \cdot \frac{dy}{dx} = y$

$x\,dy = y\,dx$

$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + C$

$y = \pm e^{\ln|x| + C}$

$y = \pm e^{\ln|x|} \cdot e^{C}$

$y = C\cancel{e^{\ln}}x$

$y = Cx$

$\boxed{2}$ a) $y \cdot y' = 6\cos(\pi x)$

$y \cdot \frac{dy}{dx} = 6\cos(\pi x)$

$\int y\,dy = \int 6\cos(\pi x)\,dx$

$u = \pi x$

$du = \pi\,dx$

$\frac{1}{\pi}du = dx$

$2 \cdot \frac{y^2}{2} = \left(\frac{6}{\pi} \cdot \sin(\pi x) + C\right) \cdot 2$

$y^2 = \frac{12}{\pi}\sin(\pi x) + C$

$y = \pm\sqrt{\frac{12}{\pi}\sin(\pi x) + C}$

b) $\sqrt{x^2-9}\cdot y' = 5x$

$\sqrt{x^2-9}\cdot\frac{dy}{dx} = 5x$ $\quad u=x^2-9$

$\int dy = \int \frac{5x}{\sqrt{x^2-9}}dx$ $\quad du=2xdx$ $\quad \frac{1}{2}du=xdx$

$y=\frac{5}{2}\int u^{-1/2}du$

$y=\frac{5}{\cancel{2}}\cdot\cancel{2}(x^2-9)^{1/2}+C$

$y=5(x^2-9)^{1/2}+C$

c) $4y\cdot y'-3e^x=0$

$4y\cdot\frac{dy}{dx}=3e^x$

$\int 4y\,dy=\int 3e^x dx$

$\frac{2y^2}{2}=\frac{3e^x+C}{2}$

$y^2=\frac{3e^x+C}{2}$

$y=\pm\sqrt{\frac{3e^x+C}{2}}$

$\boxed{3}$ a) $\sqrt{x}+\sqrt{y}\cdot y'=0$ $\quad y(1)=4$

$\sqrt{x}+\sqrt{y}\cdot\frac{dy}{dx}=0$

$\sqrt{y}\frac{dy}{dx}=-\sqrt{x}$

$\int y^{1/2}dy=-\int x^{1/2}dx$

$\frac{3}{2}\cdot\frac{2}{3}y^{3/2}=\left(-\frac{2}{3}x^{3/2}+C\right)\frac{3}{2}$

$(y^{3/2})^{2/3}=(-x^{3/2}+C)^{2/3}$

$y=(-x^{3/2}+C)^{2/3}$

$4=(-1^{3/2}+C)^{2/3}$

$\pm[4]^{3/2}=[(-1+C)^{2/3}]^{3/2}$

$\pm 8=-1+C$

$C=9,-7$ $\quad y=(-x^{3/2}+9)^{2/3}$ or $y=(-x^{3/2}-7)^{2/3}$

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b) $2xy' - \ln x^2 = 0 \qquad y(1) = 2$

$2x \cdot \frac{dy}{dx} - \ln x^2 = 0$

$2x \frac{dy}{dx} = \ln x^2$

$u = x^2$
$du = 2x\,dx$

$\frac{2x\,dy}{2x} = \frac{\ln x^2\,dx}{2x}$

$\int dy = \int \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\ln x}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$y = \int u\,du$

$y = \frac{(\ln x)^2}{2} + C$

$2 = \frac{(\ln 1)^2}{2} + C$

$2 = 0 + C$

$C = 2$

$y = \frac{(\ln x)^2}{2} + 2$

c) $y\sqrt{1-x^2} \cdot y' - x\sqrt{1-y^2} = 0$
$y(0) = 1$

$y\sqrt{1-x^2} \cdot \frac{dy}{dx} - x\sqrt{1-y^2} = 0$

$y\sqrt{1-x^2} \cdot \frac{dy}{dx} = x\sqrt{1-y^2}$

$u = 1-y^2$
$du = -2y\,dy$
$-\frac{1}{2}du = y\,dy$

$\int \frac{y}{\sqrt{1-y^2}}\,dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$

$-\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\int u^{-1/2}du$

$-\frac{1}{\cancel{2}} \cdot \cancel{2}(1-y^2)^{1/2} = -\frac{1}{\cancel{2}} \cdot \cancel{2}(1-x^2)^{1/2} + C$

$-\sqrt{1-y^2} = -\sqrt{1-x^2} + C$

$-\sqrt{1-1} = -\sqrt{1-0} + C$

$0 = -1 + C$

$C = 1$

$\sqrt{1-y^2} = \sqrt{1-x^2} + 1$

$1-y^2 = (\sqrt{1-x^2} - 1)^2$

$\sqrt{y^2} = \sqrt{-[(\sqrt{1-x^2}-1)^2 - 1]}$

$y = \pm\sqrt{-[(\sqrt{1-x^2}-1)^2 - 1]}$

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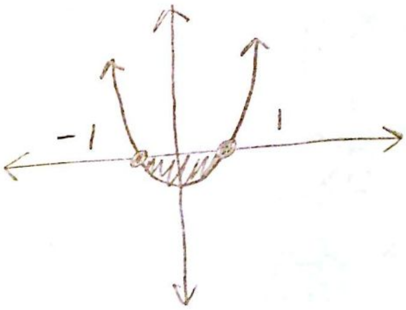
$\boxed{4}$ a) x-axis ($y=0$)

$y=x^2-1$



$$\int_{-1}^{1} (0 - x^2+1)\,dx$$

$$= \int_{-1}^{1} (-x^2+1)\,dx = -\frac{x^3}{3} + x \Big|_{-1}^{1}$$

$$\left(-\frac{1}{3}+1\right) - \left(\frac{1}{3}-1\right) = -\frac{2}{3}+2 = \frac{4}{3} \text{ units}^2$$

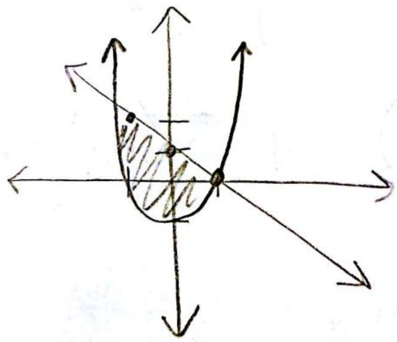
b) $y = x^2-1$, $y=-x+1$

$x^2-1 = -x+1$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2, 1$



$$\int_{-2}^{1} (-x+1-x^2+1)\,dx = \int_{-2}^{1} (-x-x^2+2)\,dx$$

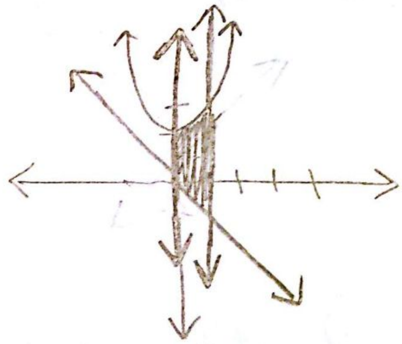
$$-\frac{x^2}{2} - \frac{x^3}{3} + 2x \Big|_{-2}^{1} = \left(-\frac{1}{2}-\frac{1}{3}+2\right) - \left(-2+\frac{8}{3}-4\right)$$

$$-\frac{3}{6} - \frac{2}{6} + \frac{12}{6} + \frac{12}{6} - \frac{16}{6} + \frac{24}{6}$$

$$\frac{27}{6} = \frac{9}{2} \text{ units}^2$$

) $y=x^2+2, \quad y=-x, \quad x=0, \quad x=1$

$$\int_0^1 (x^2+2+x)\,dx$$



$$= \frac{x^3}{3} + 2x + \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1}{3} + 2 + \frac{1}{2} = \frac{2}{6} + \frac{12}{6} + \frac{3}{6} = \frac{17}{6} \text{ units}^2$$

$\boxed{5}$

a) $\frac{dy}{dx} = xy^2$

$$\int \frac{1}{y^2}\,dy = \int x\,dx$$

$$y\left(-\frac{1}{y}\right) = \left(\frac{x^2}{2}+c\right)y$$

$$-1 = \left(\frac{x^2}{2}+c\right)\cdot y$$

$$y = \left(\frac{-1}{\frac{x^2}{2}+c}\right) 2 \qquad \frac{-2}{x^2+c}$$

$$1 = \frac{-1}{0+c}$$

$$c = -1$$

b) $y = \left(\frac{-1}{\frac{x^2}{2}-1}\right) 2$

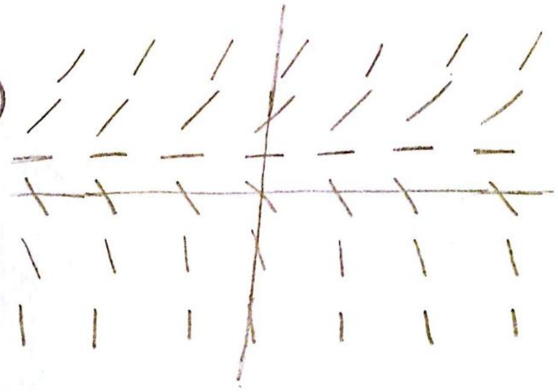
$$y = \frac{-2}{x^2-2}$$

c) $x^2+c=0 \qquad x = \pm\sqrt{c}$

$x^2 = -c$

$x^2 = c$

$\boxed{5}$ a)



c) $5y-5=0$

$y=1$

b) $\frac{dy}{dt} = 5y-5$

$\frac{1}{5y-5}\,dy = dt$

$u = y-1$

$du = dy$

$\frac{1}{5(y-1)}\,dy = dt$

$\frac{1}{5}\int \frac{1}{y-1}\,dy = \int dt$

$\frac{1}{5}\left(e^{5t} + \ln 20\right) + 1$

$\frac{1}{5}e^{5t} + \frac{1}{5}\ln 20 + 1$

$\frac{1}{5}\int \frac{1}{u}\,du = t + c$

$\cancel{5 \cdot \frac{1}{5}} \ln|y-1| = (t+c)5$

$\pm e^{5t+c} = y-1$

$Ce^{5t} + 1 = y$

$Ce^{0} + 1 = 5$

$C = 4$

$y = 4e^{5t} + 1$

1) $y - 7 = -25(x - 1)$

2) 1

3) $2\sqrt{3}$

4) $2e^{2x} + 8x$

5) $\frac{10}{\sqrt{3}} \tan^{-1}\left(\frac{x+4}{\sqrt{3}}\right) + C$