AP CALCULUS AB **Problem Set Unit 3** **Name:** ____________________________

1. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

2. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

3. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

4. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

5. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only

(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

6. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

7. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

8. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

9. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x = 3$.
(E) The graph of f intersects both axes.

10. Given the function defined by $f(x)=3x^5-20x^3$, find all values of x for which the graph of f is concave up.

(A) $x>0$

(B) $-\sqrt{2}<x<0$ or $x>\sqrt{2}$

(C) $-2<x<0$ or $x>2$

(D) $x>\sqrt{2}$

(E) $-2<x<2$

11. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S=4\pi r^2 \text{ and } V=\frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

12. If $\tan(xy)=x$, then $\frac{dy}{dx}=$

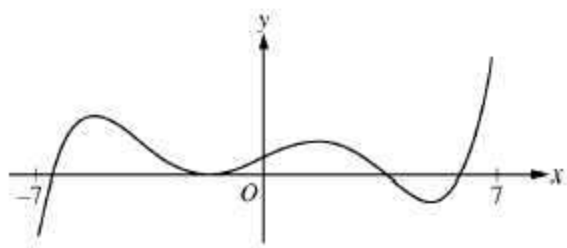
(A) $\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy)-y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy)-y}{x}$

13. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

14.



Graph of f'

The figure above shows the graph of f', the derivative of the function f, on the open interval $-7<x<7$. If f' has four zeros on $-7<x<7$, how many relative maxima does f have on $-7<x<7$?

(A) One (B) Two (C) Three (D) Four (E) Five

15. Let f be the function with first derivative defined by $f'(x)=\sin\left(x^3\right)$ for $0\le x\le 2$. At what value of x does f attain its maximum value on the closed interval $0\le x\le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

16. A company needs to construct a cylindrical container that will hold 100cm^3. The cost for the top and bottom of the can is 3 times the cost for the sides. What dimensions are necessary to minimize the cost.

(A) r=5.304, h=1.13
(B) r=6.512, h=9.465
(C) r=1.744, h=10.464
(E) r=1.744, h=18.228
(D) r=6.402, h=2.354

AP CALCULUS AB **Problem Set Unit 2** **Name:** ____________________

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

a)
b)
c)

ANSWERS: 1969 #5, 7, 9, 11, 17, 19,21, 24, 30, 1973 #22,26, 40, 2012 #24,87, 92, website

1. E	2. D	3. C	4. B	5. B	6. C	7. B	8. C
9. E	10. B	11. E	12. E	13. D	14. A	15. C	16. C

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

a)

$$f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$$

$$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$$

$2 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \end{cases}$

b)

$$f'(1) = \frac{1}{2}k - 1 = 0 \Rightarrow k = 2$$

When $k = 2$, $f'(1) = 0$ and $f''(1) = -\frac{1}{2} + 1 > 0$.

f has a relative minimum value at $x = 1$ by the Second Derivative Test.

$4 : \begin{cases} 1 : \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1 : \text{solves for } k \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

c)

At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$

$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

Therefore, $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$

$\Rightarrow 4 = \ln x$

$\Rightarrow x = e^4$

$\Rightarrow k = \frac{4}{e^2}$

$3 : \begin{cases} 1 : f''(x) = 0 \text{ or } f(x) = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{answer} \end{cases}$