Problem Set U3 Solutions:

1) $3x^2 + 2xy + y^2 = 2$

E $6x + 2x\frac{dy}{dx} + y \cdot 2 + 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-6x - 2y}{2x + 2y}$$

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-6 + 2}{2 - 2}$$ undefined

When $x = 1$,

$3(1)^2 + 2(1)y + y^2 = 2$

$3 + 2y + y^2 = 2$

$y^2 + 2y + 1 = 0$

$(y+1)^2 = 0$

$y = -1$

2) $y = x + Kx^{-1}$

D $y' = 1 - Kx^{-2} = 0$

$$1 - \frac{K}{x^2} = 0$$

$$\frac{x^2 - K}{x^2} = 0$$

$(x + \sqrt{K})(x - \sqrt{K}) = 0$

$-2 + \sqrt{K} = 0$ $\quad -2 - \sqrt{K} = 0$ X

$\sqrt{K} = 2$

$K = 4$

3) $A = \pi r^2$

C $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2 \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$

$r = \frac{1}{\pi}$

4) $x^2 + 2y = 0$ $\quad d = \sqrt{(x-0)^2 + (y+\frac{1}{2})^2}$

$y = -\frac{1}{2}x^2$ $\quad d = \sqrt{x^2 + (-\frac{1}{2}x^2 + \frac{1}{2})^2}$

B $\quad d = x^2 + (-\frac{1}{2}x^2 + \frac{1}{2})^2$

$\frac{dd}{dx} = 2x + 2(-\frac{1}{2}x^2 + \frac{1}{2}) \cdot (-x)$

$\frac{dd}{dx} = 2x - 2x(-\frac{1}{2}x^2 + \frac{1}{2})$

$2x + x^3 - x$

$x^3 + x = 0$

$x(x^2+1) = 0$

$x = 0 \qquad y = -\frac{1}{2}(0)^2$

$y = 0$

5) $y = 5x^4 - x^5$

$y' = 20x^3 - 5x^4$

B $y'' = 60x^2 - 20x^3 = 0$

$20x^2(3-x) = 0$

$x = 0, 3$

PoI : (3, 162)

6) $v = t^{-1} \cdot \ln t$

C $a = t^{-1} \cdot \frac{1}{t} + \ln t \cdot -t^{-2}$

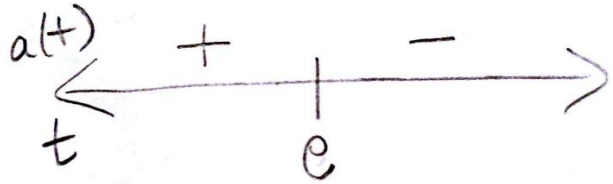
$a = \frac{1}{t^2} - \frac{\ln t}{t^2} = \frac{1 - \ln t}{t^2} = 0$

$1 - \ln t = 0$

$\ln t = 1$

$e^1 = t$

$t = e$



7) $y = x^2 + e^{-2x}$

B $y' = 2x + e^{-2x} \cdot -2$

$y' = 2x - 2e^{-2x}$

$y'(0) = 0 - 2e^0$

$0 - 2 = -2$

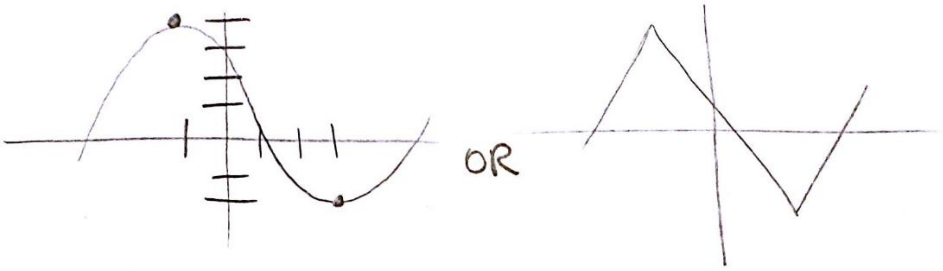
decreasing

8) $\sin x = e^y$

C $\cos x = e^y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{e^y} = \frac{\cos x}{\sin x} = \cot x$

9)

E 

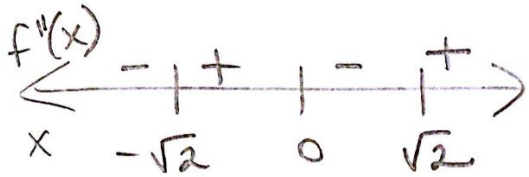
10) $y = 3x^5 - 20x^3$

B $y' = 15x^4 - 60x^2$

$y'' = 60x^3 - 120x = 0$

$60x(x^2 - 2) = 0$

$x = 0, \pm\sqrt{2}$



11) $\frac{dr}{dt} = .3$ in/s

E $S = 100\pi = 4\pi r^2$

$r = 5$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(5)^2(.3)$

30π

12) $\tan(xy) = x$

$\sec^2(xy) \cdot \left(x \frac{dy}{dx} + y\right) = 1$

E $x \frac{dy}{dx} + y = \frac{1}{\sec^2(xy)}$

$\frac{dy}{dx} = \frac{\frac{1}{\sec^2(xy)} - y}{x}$

13) $A = \pi r^2$ $\quad C = 2\pi r$

D $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$2 \cdot \frac{dC}{dt} = 2\pi r \cdot \frac{dC}{dt} \cdot \frac{1}{2\pi}$ $\quad \frac{dr}{dt} = \frac{dC}{dt} \cdot \frac{1}{2\pi}$

$2 = r$

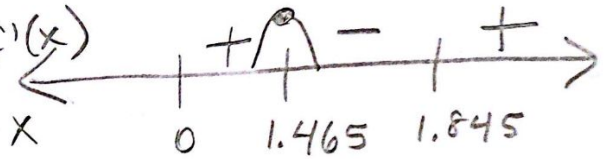
14) A b/c rel max is where $f'(x)$ goes from pos to neg

15) $f'(x) = \sin(x^3)$ $\quad 0 \le x \le 2$

$\sin(x^3) = 0$ $\quad x^3 = \pi$ $\quad x^3 = 2\pi$

C $x^3 = 0$ $\quad x = 1.465$ $\quad 1.845$

$x = 0$



16) $SA = 2\pi rh + 2\pi r^2$

C $V = \pi r^2 h = 100$

$h = \frac{100}{\pi r^2}$

$C = 3 \cdot 2\pi r^2 + 2\pi rh$

$C = 6\pi r^2 + 2\pi rh$

$C = 6\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$

$C' = 12\pi r - 200r^{-2}$

$12\pi r - \frac{200}{r^2}$

$12\pi r^3 - 200 = 0$

$12\pi r^3 = 200$

$r = 1.744$