1. $\int_0^1 \frac{x^2}{x^2+1}\,dx =$

(A) $\frac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2}\ln 2$ (E) $\frac{4+\pi}{4}$

2. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

3. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

4. $\int_0^1 \sqrt{x^2-2x+1}\,dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

5. $\int_{-1}^2 \frac{|x|}{x}\,dx$ is

(A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

6. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

7. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

8. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

9. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\, dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y} dy$ (C) $2\int_0^{\pi/4} \sin^2 y\, dy$

(D) $\int_0^{\pi/4} \sin^2 y\, dy$ (E) $2\int_0^{\pi/6} \sin^2 y\, dy$

10. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

11. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

12. $\int_1^2 \frac{x+1}{x^2+2x} dx =$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

13. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}} dx$?

(A) $\arcsin\frac{x}{5} + C$ (B) $\arcsin x + C$ (C) $\frac{1}{5}\arcsin\frac{x}{5} + C$

(D) $\sqrt{25-x^2} + C$ (E) $2\sqrt{25-x^2} + C$

14. $\int \frac{dx}{(x-1)(x+2)} =$

(A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$ (B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right| + C$ (C) $\frac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

(D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right) + C$ (E) $\ln\left|(x-1)(x+2)^2\right| + C$

15. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

16. If $\int f(x)\sin x\, dx = -f(x)\cos x + \int 3x^2 \cos x\, dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

Problem Set A ANSWERS: 1969BC#10,18,23,26, 1973BC#5,12,21,25,28,35, 1985BC#39,3,7,12,13,21

1. A	2. E	3. D	4. C	5. B	6. D	7. B	8. B
9. C	10. C	11. A	12. B	13. A	14. A	15. B	16. B

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) $\left.\frac{dW}{dt}\right\|_{t=0} = \frac{1}{25}(W(0) - 300) = \frac{1}{25}(1400 - 300) = 44$ The tangent line is $y = 1400 + 44t$. $W\left(\frac{1}{4}\right) \approx 1400 + 44\left(\frac{1}{4}\right) = 1411$ tons	$2: \begin{cases} 1: \frac{dW}{dt} \text{ at } t = 0 \\ 1: \text{answer} \end{cases}$
(b) $\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{625}(W - 300)$ and $W \geq 1400$ Therefore $\frac{d^2W}{dt^2} > 0$ on the interval $0 \leq t \leq \frac{1}{4}$. The answer in part (a) is an underestimate.	$2: \begin{cases} 1: \frac{d^2W}{dt^2} \\ 1: \text{answer with reason} \end{cases}$
(c) $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ $\int \frac{1}{W - 300}\, dW = \int \frac{1}{25}\, dt$ $\ln\|W - 300\| = \frac{1}{25}t + C$ $\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$ $W - 300 = 1100e^{\frac{1}{25}t}$ $W(t) = 300 + 1100e^{\frac{1}{25}t}, \ 0 \leq t \leq 20$	$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } W \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables

AP CALCULUS BC **Problem Set B Unit 1** **Name:** ____________________

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim\limits_{x \to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

Problem Set B ANSWERS: 1969BC#10,18,23,26, 1973BC#5,12,21,25,28,35, 1985BC#39,3,7,12,13,21

1. A	2. E	3. D	4. C	5. B	6. D	7. B	8. B
9. C	10. C	11. A	12. B	13. A	14. A	15. B	16. B

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) $\lim_{x\to 0}(f(x)+1) = -1+1 = 0$ and $\lim_{x\to 0}\sin x = 0$ Using L'Hospital's Rule, $\lim_{x\to 0}\frac{f(x)+1}{\sin x} = \lim_{x\to 0}\frac{f'(x)}{\cos x} = \frac{f'(0)}{\cos 0} = \frac{(-1)^2\cdot 2}{1} = 2$	$2: \begin{cases} 1: \text{L'Hospital's Rule} \\ 1: \text{answer} \end{cases}$
(b) $f\left(\frac{1}{4}\right) \approx f(0) + f'(0)\left(\frac{1}{4}\right)$ $= -1 + (2)\left(\frac{1}{4}\right) = -\frac{1}{2}$ $f\left(\frac{1}{2}\right) \approx f\left(\frac{1}{4}\right) + f'\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ $= -\frac{1}{2} + \left(-\frac{1}{2}\right)^2\left(2\cdot\frac{1}{4}+2\right)\left(\frac{1}{4}\right) = -\frac{11}{32}$	$2: \begin{cases} 1: \text{Euler's method} \\ 1: \text{answer} \end{cases}$
(c) $\frac{dy}{dx} = y^2(2x+2)$ $\frac{dy}{y^2} = (2x+2)\,dx$ $\int \frac{dy}{y^2} = \int (2x+2)\,dx$ $-\frac{1}{y} = x^2 + 2x + C$ $-\frac{1}{-1} = 0^2 + 2\cdot 0 + C \Rightarrow C = 1$ $-\frac{1}{y} = x^2 + 2x + 1$ $y = -\frac{1}{x^2+2x+1} = -\frac{1}{(x+1)^2}$	$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables