Problem Set 1 :

1) $\int_0^1 \frac{x^2}{x^2+1}\,dx = \int_0^1 \frac{x^2+1-1}{x^2+1}\,dx = \int_0^1 \frac{x^2+1}{x^2+1}\,dx - \int_0^1 \frac{1}{x^2+1}\,dx$

A

$= x - \tan^{-1}x \Big|_0^1 = 1 - \frac{\pi}{4} = \frac{4-\pi}{4}$

2) E $\rightarrow$ vertex is sharp turn

3) $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$

D $\int y\,dy = \int \frac{-x}{e^{x^2}}\,dx$ $\quad u = x^2$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

$\frac{y^2}{2} = -\frac{1}{2}\int e^{-u}\,du$

$\frac{y^2}{2} = +\frac{1}{2}e^{-x^2} + C$

$\frac{2^2}{2} = \frac{1}{2}e^0 + C$

$2 = \frac{1}{2} + C$

$C = \frac{3}{2}$

$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} + \frac{3}{2}$

$y^2 = \frac{1}{e^{x^2}} + 3$

$y = \sqrt{3 + e^{-x^2}}$

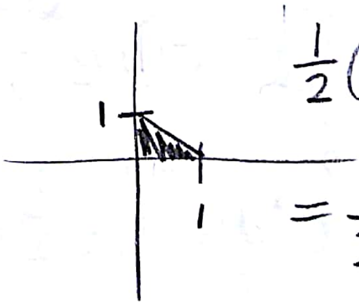
4) $\int_0^1 \sqrt{x^2-2x+1}\,dx$

C

$\sqrt{x^2-2x+1} = \sqrt{(x-1)^2}$

$= |x-1|$

$\int_0^1 |x-1|\,dx$



$\frac{1}{2}(1)(1)$

$= \frac{1}{2}$

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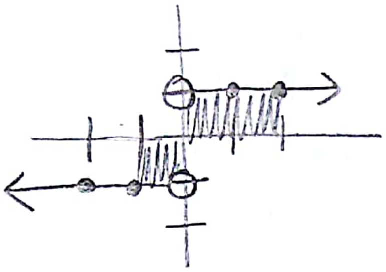
5) $\int_{-1}^{2} \frac{|x|}{x}\,dx$

B



$-1+2=1$

6) $\int_{1}^{K} x^{n-1}\,dx = \frac{1}{n}$

D

$\frac{x^n}{n}\Big|_1^K = \frac{1}{n}$

$\frac{K^n}{n} - \frac{1^n}{n} = \frac{1}{n}$

$\frac{K^n - 1^n}{n} = \frac{1}{n}$

$K^n - 1 = 1$

$(K^n)^{1/n} = (2)^{1/n}$

$K = 2^{1/n}$

7) $\int_{0}^{1} (x+1)e^{x^2+2x}\,dx$

B

$u = (x^2 + 2x)$

$du = (2x+2)dx$

$\frac{1}{2}du = (x+1)dx$

$\frac{1}{2}\int_{0}^{3} e^u\,du = \frac{1}{2}e^u\Big|_0^3$

$\frac{1}{2}e^3 - \frac{1}{2}$

8) $\int_{0}^{\frac{\pi}{4}} (\tan^2 x)\,dx$

B

$\int_{0}^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x}\,dx$

$\int_{0}^{\frac{\pi}{4}} \frac{1-\cos^2 x}{\cos^2 x}\,dx$

$\int_{0}^{\pi/4} (\sec^2 x - 1)\,dx$

$= \tan x - x \Big|_0^{\pi/4}$

$\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4}$

$1 - \frac{\pi}{4}$

C 9) $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

$\sqrt{x} = \sin y$
$x = \sin^2 y$
$dx = 2\sin y\cos y\,dy$

$\int_0^{\frac{\pi}{4}} \frac{\sin y}{\sqrt{1-\sin^2 y}} \cdot 2\sin y\cos y\,dy = \frac{2\sin^2 y \cdot \cancel{\cos y}}{\cancel{\sqrt{\cos^2 y}}}$

$= 2\int_0^{\frac{\pi}{4}} \sin^2 y\,dy$

C 10) $\pi\int_0^{\pi/4} \sec^2 x\,dx = \pi\tan x\Big|_0^{\pi/4}$

$\pi(1) - 0 = \pi$

A 11) $\int_0^3 (e^{-x})^2 dx = \int_0^3 e^{-2x} dx$

$u = -2x$
$du = -2dx$
$-\frac{1}{2}du = dx$

$-\frac{1}{2}\int_0^{-6} e^u = \frac{1}{2}\int_{-6}^{0} e^u = \frac{1}{2}e^u\Big|_{-6}^{0} = \frac{1}{2} - \frac{1}{2}e^{-6}$

B 12) $\int_1^2 \frac{x+1}{x^2+2x}\,dx$

$x(x+2)$

$\frac{x+1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$x+1 = A(x+2) + Bx$

$x=-2;\ -1 = B(-2) \quad B = \frac{1}{2}$

$x=0;\ 1 = 2A \quad A = \frac{1}{2}$

$\frac{1}{2}\int_1^2 \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x+2}dx = \frac{1}{2}\ln|x| + \frac{1}{2}\ln|x+2|\Big|_1^2$

$\frac{1}{2}(\ln\frac{2\cdot 4}{3})$ $\quad \frac{1}{2}\ln\frac{8}{3}$ $\quad = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 4 - (\cancel{\frac{1}{2}\ln 1} + \frac{1}{2}\ln 3)$

13) $\int \frac{1}{\sqrt{25-x^2}}\,dx = \arcsin\frac{x}{5} + C$

A

14) $\int \frac{1}{(x-1)(x+2)}\,dx$ $\quad \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

A

$1 = A(x+2) + B(x-1)$

$x=-2:\ 1 = B(-3)\quad B = -\frac{1}{3}$

$x=1:\ 1 = 3A\quad A = \frac{1}{3}$

$\frac{1}{3}\int \frac{1}{x-1}\,dx - \frac{1}{3}\int \frac{1}{x+2}\,dx$

$\frac{1}{3}\left(\ln|x-1| - \ln|x+2|\right) + C$

15) $f(x) = x^3 - 3x^2$

B

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = \cancel{0}, 2$

$\frac{f(b)-f(a)}{b-a} = \frac{0-0}{3-0} = 0$

$f(3) = 3^3 - 3(3)^2 = 0$

$f(0) = 0$

open interval $(0,3)$

16) $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx$

B

$u = f(x)\qquad v = -\cos x$

$du = f'(x)dx\qquad dv = \sin x\,dx$

$f'(x) = 3x^2$

$f(x) = x^3$

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