Problem Set 5 – Logarithmic Functions
AP Calculus AB

Name: ______________________

Date: ______________________

Non-Calculator

1) The inverse of $y = \ln x - 1$ is
(A) $y = e^{x-1}$
(B) $y = e^x - 1$
(C) $y = e^x + 1$
(D) $y = e^{x+1} - 1$
(E) $y = e^{x+1}$

2) Find the derivative of $y = 6\ln\left(\frac{1}{x^2}\right)$.
(A) -6
(B) $-\frac{12}{x^3}$
(C) $\frac{6}{x}$
(D) $-\frac{12}{x}$
(E) $-\frac{12}{x^2}$

3) Which of the following statements is (are) false for $f(x) = e^x sinx$?
I. $\lim_{x\to 0} f(x) = 0$
II. $\lim_{x\to 0} f'(x) = 1$
III. $\lim_{x\to 0} f''(x) = 2$
(A) I only
(B) II only
(C) III only
(D) II and III only
(E) None of the statements is false

4) If $y = \ln(\tan x)$, then $y' =$
(A) $\frac{2}{\sin 2x}$
(B) $\sec^2 x$
(C) $\frac{1}{x \tan x}$
(D) $\cot x$
(E) $\sec^2 x \tan x$

5) If $y = e^{5x+5}$, then $y'(0) =$
(A) e^5
(B) 1
(C) $5e^5$
(D) 5
(E) $\frac{1}{5}e^5$

6) If $y = \ln e^{\tan^2 x}$, find $y'\left(\frac{\pi}{4}\right)$
(A) -2
(B) 1
(C) 2
(D) $2\sqrt{2}$
(E) 4

7) $(\arctan 3x)' =$
(A) $\frac{3}{1+3x^2}$
(B) $\frac{3}{1+x^2}$
(C) $\frac{3}{1+9x^2}$
(D) $\frac{1}{1+9x^2}$
(E) $\frac{3x}{1+3x^2}$

8) $\frac{d}{dx}(\arcsin(x^2)) =$
(A) $\frac{x^2}{\sqrt{1-x^4}}$
(B) $\frac{2x}{\sqrt{1-x^4}}$
(C) $\frac{2x}{\sqrt{1-x^2}}$
(D) $\frac{1}{\sqrt{1-x^4}}$
(E) $\frac{4x}{\sqrt{1-x^2}}$

9) The second derivative of $f(x) = \ln(x)$ at $x = 3$ is
(A) $-\frac{1}{3}$
(B) $-\frac{1}{9}$
(C) $\frac{1}{9}$
(D) $\frac{1}{3}$
(E) $\frac{2}{3}$

10) Find the equation of the line perpendicular to the line tangent to $f(x) = \ln(3-2x)$ at $x = 1$.
(A) $y = -2x + 1$
(B) $y = \frac{1}{2}x + 1$
(C) $y = \frac{1}{2}(x-1)$
(D) $y = \frac{1}{2}(x+1)$
(E) $y = -2x + 2$

Problem Set 5 – Logarithmic Functions Name: ______________________
AP Calculus AB Date: ______________________

11) Find the derivative of $y = e^x \sin x$. (A) $e^x \cos x$ (B) $e^x + \cos x$ (C) $e^x(\sin x + \cos x)$ (D) $\ln(\sin x)$ (E) $e \cos x$	17) The function $f(x) = x^5 + 3x - 2$ passes through the point (1, 2). Let f^{-1} denote the inverse of f. Then $(f^{-1})'(2)$ equals (A) 1/83 (B) 1/8 (C) 1 (D) 8 (E) 83
12)Find the average rate of change of $y = \ln(x^2)$ on the interval [1, 2] (A) –ln 4 (B) 0 (C) ln 4 (D) 2 (E) 4	18) The function $f(x) = e^x - x + 2$ has (A) a relative minimum at (0, 3) (B) a relative minimum at (0, 0) (C) a relative maximum at (0, 3) (D) two critical values (E) a relative minimum at (0, 0) and a relative maximum at (0, 3)
13) Find the number of horizontal asymptotes of $y = 2 - \ln x$. (A) 0 (B) 1 (C) 2 (D) 3 (E) 4	19) On what interval is the graph of $f(x) = \ln(x^2 + 1)$ concave up? (A) (0, 1) (B) (−1, 1) (C) (−0.5, 0.5) (D) $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$ (E) The graph is never concave up.
14) $f(x) = \ln(\sin x)$. Find $f'\left(\frac{\pi}{4}\right)$. (A) $-\frac{1}{2}\ln 2$ (B) $\frac{\sqrt{2}}{2}$ (C) 0 (D) 1 (E) undefined	20) If $y = 4^{x^2}$, what is y′(1)? (A) 0 (B) ln 4 (C) 2 ln 4 (D) 1 + 2 ln 4 (E) 8 ln 4
15) $y = \ln(e^{x^2-1})$. Find $y'(1)$. (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined	21) $\int \frac{e^{2x} - e^{3x}}{e^x}\,dx =$ (A) –1 (B) $\frac{1}{2}e^{-x} + c$ (C) $e^{-2x} + c$ (D) $e^x - 2e^{2x} + c$ (E) $e^x - \frac{1}{2}e^{2x} + c$
16) FREE SPACE	22) $\int_0^{\frac{\pi}{2}} e^{2-\cos x} \sin x\,dx =$ (A) $e^2 - e$ (B) 1 (C) 0 (D) e^2 (E) does not exist

Problem Set 5 – Logarithmic Functions Name: ______________________
AP Calculus AB Date: ______________________

23)

What is the instantaneous rate of change of $f(x) = \ln(\tan^2 x)$ at $x = \frac{\pi}{4}$?

(A) 0
(B) 1
(C) $\frac{\sqrt{3}}{2}$
(D) 4
(E) undefined

24)

$f(x) = e^{\sin^2 x}$. $f'(x) =$

(A) $e^{\sin^2 x}$
(B) $2 \sin x e^{\sin^2 x}$
(C) $2 \sin x \cos x e^{\sin^2 x}$
(D) $e^{2\cos x}$
(E) $e^{\cos^2 x}$

25)

If $v(t) = \ln(t^2 + t + 1)$, then $a(1) =$

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 1
(D) $\frac{4}{3}$
(E) 3

26)

$y = \frac{e^{2x-1}}{x}$ has

I a relative minimum at $x = \frac{1}{2}$
II a horizontal asymptote $y = 0$
III a vertical asymptote $x = 0$

(A) I only
(B) I and II
(C) I and III
(D) II and III
(E) I, II, and III

27)

If y = ln(4x + 1), then $\frac{d^2y}{dx^2}$ is

(A) ¼
(B) $\frac{-1}{(4x+1)^2}$
(C) $\frac{-4}{(4x+1)^2}$
(D) $\frac{-16}{(4x+1)^2}$
(E) $\frac{-1}{16(4x+1)^2}$

28)

$\int \frac{dx}{1+4x^2}$

(A) $\tan^{-1}(2x)+C$
(B) $\frac{1}{8}\ln(1+4x^2)+C$
(C) $\frac{1}{8(1+4x^2)}+C$
(D) $\frac{1}{2}tan^{-1}(2x)+C$
(E) $\frac{1}{8x}\ln|1+4x^2|+C$

29)

The derivative of $y(x) = \arcsin \frac{x}{2}$ on $-1 < x < 1$ is

(A) $y = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$
(B) $y = \frac{1}{2\sqrt{1-\sin(x)}}$
(C) $y = \frac{1}{2\cos\left(\arcsin\frac{x}{2}\right)}$
(D) $y = \frac{\arccos\frac{x}{2}}{2}$
(E) $y = \frac{\arccos\frac{x}{2}}{2}$

30) $\int \frac{dx}{x^2+2x+2}$

(A) $\ln(x^2 + 2x + 2)$
(B) $\ln|x + 1| + C$
(C) $\arctan(x + 1) + 3$
(D) $\frac{1}{\frac{1}{3}x^3+x^2+2x}+C$
(E) $-\frac{1}{x}+\frac{1}{2}\ln|x|+\frac{x}{2}+C$

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Free Response (Non-Calculator)

1. E	2. D	3. E	4. A	5. C	6. E
7. C	8. B	9. B	10. C	11. C	12. C
13. A	14. D	15. D	16. omit	17. B	18. A
19. B	20. E	21. E	22. A	23. D	24. C
25. C	26. E	27. D	28. D	29. A	30. C

a) 1 point

$v(t) = y'(t) = 2t - \frac{4}{t+1}$

b) 4 points

1 point setting the derivative equal to zero
1 point for x-value
1 point for y- value
1 point for justification

$2t - \frac{4}{t+1} = 0$; t = -2 and t = 1. Only t = 1 is positive and therefore the correct answer.

T = 1 and y (1) = -4ln2

T = 1 is a local minimum by the first derivative test. Since there are no other critical points to t >0, y(1) is the global minimum.

c) 2 points

1 point for answer
1 point for justification

The speed of the particle is increasing when a(t) and v(t) have the same sign. The speed of the particle is increasing for t > 1.

d) 2 points

1 point for answer
1 point for considering the change in direction at t = 1

$|-4\ln 2 + 1| + |3 - 4\ln 3 + 4\ln 2|$