Prob Set Unit 2-3

1) $n^x = \frac{1}{n^{-x}}$ $\quad -x > 1$ $\quad$ by p-series

B $\quad x < -1$

2) $f(x) = (x^2+1)^{(2-3x)}$ $\quad f'(1)$

A $\ln y = (2-3x)\ln(x^2+1)$

$$\frac{1}{y}\frac{dy}{dx} = (2-3x)\cdot\frac{2x}{x^2+1} + \ln(x^2+1)(-3)$$

$$\frac{dy}{dx} = y\left[(2-3x)\cdot\frac{2x}{x^2+1} - 3\ln(x^2+1)\right]$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 2^{-1}\left[(-1)(1) - 3\ln 2\right]$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{2}(-1-3\ln 2)$$

$$\frac{1}{2}(-1-\ln 8)$$

$$-\frac{1}{2}(1+\ln 8)$$

$$-\frac{1}{2}(\ln e + \ln 8) = -\frac{1}{2}(\ln 8e)$$

3) E $\quad \lim_{k\to\infty}\left|\frac{(x+1)^{k+1}}{(k+1)^2}\cdot\frac{k^2}{(x+1)^k}\right|$

$$\lim_{k\to\infty}\left|\frac{k^2}{(k+1)^2}\cdot(x+1)\right|$$

$\frac{k^2}{(k+1)^2} = 1$

$$|x+1| < 1$$

$$-1 < x+1 < 1$$

$$-2 < x < 0$$

$x = -2$: $\frac{(-1)^k}{k^2}$ $\quad$ Conv, AST

$x = 0$: $\frac{1^k}{k^2} = \frac{1}{k^2}$ $\quad$ Conv, p

IOC: $-2 \le x \le 0$

4) D
I. conv, p
II. div, p
III. conv, AST

5) C
I. $\frac{1}{n^2}$ conv, p
II. div, p
III. conv, AST

6) D
$\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right|$

$\lim_{n\to\infty} \left| \frac{n}{n+1} \cdot x-1 \right|$

$|x-1| < 1$

$-1 < x-1 < 1$

$0 < x < 2$

$x=0: \frac{(-1)^n}{n}$ conv AST

$x=2: \frac{1^n}{n}$ div, p

IOC: $0 \le x < 2$

7) C
geo $\frac{a}{1-r}$ $\frac{\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}} = \frac{\left(\frac{1}{3}\right)^n}{\frac{2}{3}} = \frac{3}{2} \cdot \left(\frac{1}{3}\right)^n$

8) A
I. conv AST
II. $\frac{1}{n}\left(\frac{3}{2}\right)^n = \frac{3^n}{n \cdot 2^n}$ div n^{th}
III. $\frac{1}{n \ln n}$ $\int_2^\infty \frac{1}{x \ln x} dx$ $u = \ln x$ $du = \frac{1}{x} dx$ $\int \frac{1}{u} du$

div integral $\lim_{b\to\infty} \ln(\ln x) \Big|_2^b = \ln(\ln b) - \ln(\ln 2)$

9) $\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ $\quad u = 9-x^2$ $\quad du = -2x\,dx$ $\quad \int u^{-1/3}\,du$

E

$\frac{3}{2}u^{2/3}$ $\quad \lim_{b\to\infty} \frac{3}{2}(9-x^2)^{2/3}\Big|_4^b$

$\frac{3}{2}(9-x^2)^{2/3}$ $\quad \lim_{b\to\infty}\left(\frac{3}{2}(9-b^2)^{2/3} - \frac{3}{2}(9-16)^{2/3}\right)$

diverges

10) $x(t) = \sin(2t) - \cos(3t)$

$v(t) = x'(t) = 2\cos(2t) + 3\sin(3t)$

E $\quad a(t) = -4\sin(2t) + 9\cos(3t)$

$-4\sin(2\pi) + 9\cos(3\pi)$

$= 0 + 9(-1)$

$= -9$

11) I. conv, DCT

A II. conv, geo

III. conv, AST

12) $\int x\cdot\sec^2 x\,dx$ $\quad u = x$ $\quad du = dx$ $\quad v = \tan x$ $\quad dv = \sec^2 x\,dx$

E

$x\tan x - \int \tan x\,dx$

$x\tan x + \ln|\cos x| + C$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$-\int \frac{1}{u}\,du$ $\quad u = \cos x$ $\quad du = -\sin x\,dx$ $\quad -du = \sin x\,dx$

$-\ln|u|$

$-\ln|\cos x|$

13) $\lim_{n\to\infty} S_n = \frac{5^n}{5^{n+1}} \to \frac{5^n}{5^n\cdot 5} \to \frac{1}{5}$

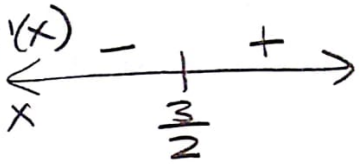
A

14) $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$

C $f'(x) = e^{(x^2-3x)^2} \cdot (2x-3) = 0$

$2x-3=0$

$x = \frac{3}{2}$



15) $\lim_{x\to 0} (1+2x)^{\csc x}$

E $\ln y = \lim_{x\to 0} \csc x \cdot \ln(1+2x)$

$\ln y = \lim_{x\to 0} \frac{\ln(1+2x)}{\sin x}$ $\frac{0}{0}$ ∴ L'H

$\ln y = \lim_{x\to 0} \frac{\frac{2}{1+2x}}{\cos x}$

$\ln y = 2$

$y = e^2$

16) $f(1) = (\sin^2(1))^K$ geo $\frac{a}{1-r}$ $\frac{\sin^2(1)}{1-\sin^2(1)}$

*calc

2.426

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