

AP CALCULUS AB PRACTICE TEST 1
Section I, Part A: Multiple-Choice Questions
Time: 55 minutes
Number of Questions: 28

A calculator may not be used on this part of the examination.

1. What is $\int \frac{x-3}{x} dx$?

- (A) $1 - 3 \ln x + C$
- (B) $x - 3 \ln x + C$
- (C) $1 + \frac{3}{x^2} + C$
- (D) $\frac{x^2 - 3x}{x^2} + C$
- (E) $\frac{x^2}{2} - 3 \ln x + C$

2. What is the value of $\lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{x^2 - 1}$?

- (A) $\frac{5}{2}$
- (B) 1
- (C) 0
- (D) $-\frac{5}{2}$
- (E) The limit does not exist.

3. What is the equation of the tangent to $f(x) = 3x - 5 \cos 2x$ at $x = 0$?

- (A) $y = -5x - 3$
- (B) $y = -5x + 3$
- (C) $y = 3x + 5$
- (D) $y = 3x - 5$
- (E) $y = x - 5$

4. A particle moves along the y -axis so that its position at any time t , for $0 \leq t \leq 5$, is given by $y(t) = t^4 - 18t^2$.

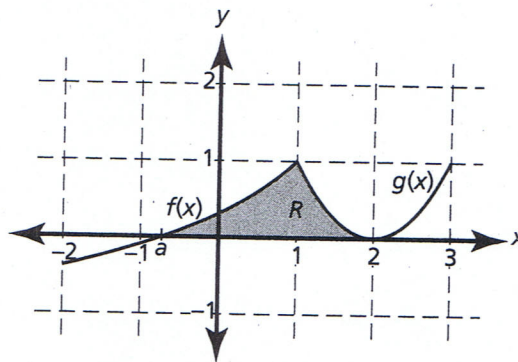
In which interval(s) is the particle speeding up?

- (A) $0 < t < \sqrt{3}$
- (B) $0 < t < \sqrt{3}$ and $3 < t < 5$
- (C) $3 < t < 5$
- (D) $\sqrt{3} < t < 3$ and $3 < t < 5$
- (E) $\sqrt{3} < t < 3$

5. Which of the following statements is (are) false for $f(x) = e^x \sin x$?

- I. $\lim_{x \rightarrow 0} f(x) = 0$
- II. $\lim_{x \rightarrow 0} f'(x) = 1$
- III. $\lim_{x \rightarrow 0} f''(x) = 2$

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) None of the statements is false.



6. The region R , bounded by $f(x)$, $g(x)$, and the x -axis, is shown in the diagram above. Which one of the following integrals represents the volume of the solid generated when R is rotated about the line $y = 1$?

- (A) $\pi \int_a^1 [1 - f(x)]^2 dx + \pi \int_1^2 [1 - g(x)]^2 dx$
- (B) $\pi \int_a^1 \{1^2 - [1 - f(x)]^2\} dx + \pi \int_1^2 \{1^2 - [1 - g(x)]^2\} dx$
- (C) $\pi \int_a^1 \{1 - [f(x)]^2\} dx + \pi \int_1^2 \{1 - [g(x)]^2\} dx$
- (D) $\pi \int_a^1 [1 - f(x^2)] dx + \pi \int_1^2 [1 - g(x^2)] dx$
- (E) none of these

7. Let $f(x) = (3 + 2x - x^2)^3$ be defined for the closed interval $-2 \leq x \leq 3$. If M is the y -coordinate of the absolute maximum and m is the y -coordinate of the absolute minimum, what is $|M + m|$?
- (A) 189
(B) 125
(C) 64
(D) 61
(E) none of these

8. Find the equation of the curve that passes through the point $(1, 2)$ and has a slope of $\left(3 + \frac{1}{x}\right)y$ at any point (x, y) on the curve.
- (A) $2xe^{3x-3}$
(B) $2xe^{3x+3}$
(C) $2xe^3$
(D) $2e^{3x-3}$
(E) none of these

9. A continuous function $h(t)$ is defined in the closed interval $10 \leq t \leq 16$ with values given in the table below. Using the data, find the trapezoidal approximation with three subintervals of unequal length to estimate $\int_{10}^{16} h(t) dt$.

t	$h(t)$
10	10
12	20
15	50
16	80

- (A) $\frac{359}{3}$
(B) 130
(C) 200
(D) 270
(E) 718

10. Find the x -coordinate of the point on $f(x) = \frac{4}{\sqrt{x}}$ that is closest to the origin.
- (A) 1
(B) 2
(C) $\sqrt{2}$
(D) $2\sqrt{2}$
(E) $\sqrt[3]{2}$

11. Evaluate $\lim_{x \rightarrow \infty} \frac{3 - \sqrt{x^2 - 1}}{2x + 5}$.
- (A) $-\frac{1}{2}$
(B) 0
(C) $\frac{3}{5}$
(D) $\frac{3}{2}$
(E) The limit does not exist.

12. If $\tan y + x^3 = y^2 + 1$ and $\frac{dx}{dt} = -2$, what is the value of $\frac{dy}{dt}$ at the point $(1, 0)$?
- (A) -6
(B) -2.5
(C) 0
(D) $\frac{1}{2(\cos 1)^2}$
(E) 6

13. A particle moves in a line with velocity $v(t) = 3t^2 - e^t$. What is the average velocity of the particle in the closed interval $0 \leq t \leq 2$?
- (A) $\frac{8 - e^2}{2}$
(B) $\frac{9 - e^2}{2}$
(C) $\frac{11 - e^2}{2}$
(D) $\frac{13 - e^2}{2}$
(E) $13 - e^2$

14. What is the value of $k + c$ if

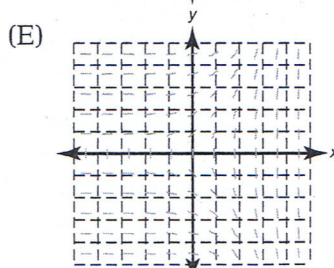
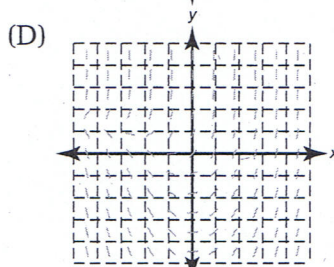
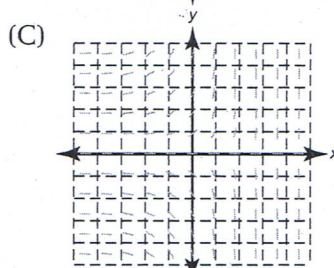
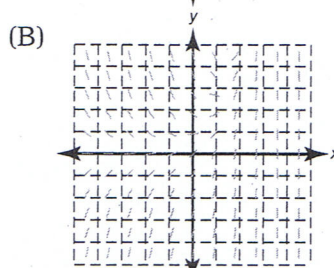
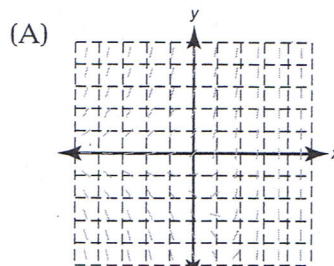
$$f(x) = \begin{cases} 2kx^2 - x, & x > 3 \\ x^3 + cx, & x \leq 3 \end{cases}$$

is everywhere differentiable?

- (A) $\frac{5}{4}$
 (B) 3
 (C) 8
 (D) 11
 (E) 24
15. A particle moves along the x -axis with a velocity given by $v(t) = t - \sqrt[3]{t}$ for $0 \leq t \leq 8$. If the particle is 4 units to the left of the origin at $t = 0$, where is the particle at $t = 8$?
- (A) 24 units to the right of the origin
 (B) 20 units to the right of the origin
 (C) 16 units to the right of the origin
 (D) $1\frac{3}{4}$ units to the left of the origin
 (E) $3\frac{1}{2}$ units to the left of the origin
16. $\int_{-2}^5 |x+1| dx =$
- (A) 17
 (B) 17.5
 (C) 18.5
 (D) 19
 (E) 19.5
17. $\int \frac{x+2}{x^2+1} dx =$
- (A) $\ln(x^2+1) + C$
 (B) $\left(\frac{x^2}{2} + 2x\right) \ln(x^2+1) + C$
 (C) $\frac{1}{2} \ln(x^2+1) + C$
 (D) $\frac{1}{2} \ln(x^2+1) + 2 \tan^{-1}(x^2) + C$
 (E) $\frac{1}{2} \ln(x^2+1) + 2 \tan^{-1}(x) + C$
18. The tangent line to the graph of $g(x)$ at the point $(3, 5)$ has a slope of -2 . Use the equation of the tangent to estimate $g(2.98)$.
- (A) 2.50
 (B) 4.98
 (C) 5.02
 (D) 5.04
 (E) 7.02

19. Which of the following is the slope

field for $\frac{dy}{dx} = \frac{e^x}{y}$?



20. Consider the piecewise function,

$$g(x) = \begin{cases} e^{\left(\frac{x}{10} + C\right)}, & x < 10 \\ 3, & x = 10 \\ \log(x) + 1, & x > 10 \end{cases}$$

Find the value of C so that $\lim_{x \rightarrow 10} g(x)$ exists.

- A) 3
 B) 2
 C) 1
 D) $\ln 2 - 1$
 E) The limit does not exist.

21. If $y = 4^{x^2}$, what is $y'(1)$?

- (A) 0
 (B) $\ln 4$
 (C) $2 \ln 4$
 (D) $1 + 2 \ln 4$
 (E) $8 \ln 4$

22. What is the value of $g(2)$ if

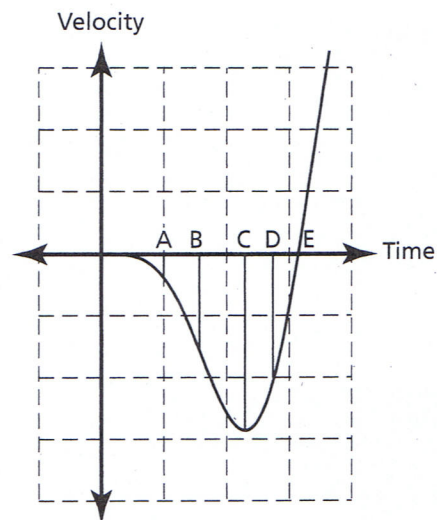
$$g(x) = 3 + \frac{d}{dx} \left[\int_1^{x^2} (1+t^2) dt \right] ?$$

- (A) 8
 (B) 20
 (C) 23
 (D) 24
 (E) 71

23. Which of the following statements is

true for $f(x) = \frac{1+e^x}{e^x-1}$?

- (A) $f(x)$ has a relative maximum at $x = 1$.
 (B) $f(x)$ has a y -intercept at $x = 0$.
 (C) $f(x)$ has a root of 0.
 (D) $f(x)$ is decreasing for all x , $x \neq 0$.
 (E) $f(x)$ has a vertical asymptote at $x = 1$.



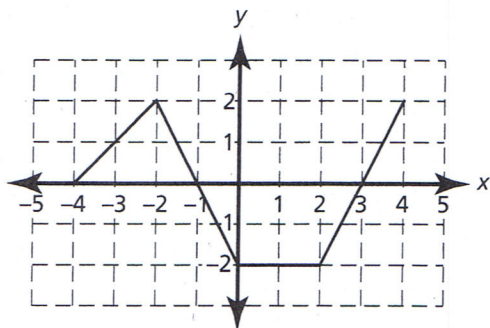
24. A particle moves along the x -axis. At which time on the velocity versus time graph given above is the particle farthest left of its starting point?

- (A) A
 (B) B
 (C) C
 (D) D
 (E) E

25. If $\sin xy = x + y$, what is $\frac{dy}{dx}$?

- (A) $\frac{y \cos xy + 1}{1 - x \cos xy}$
 (B) $\frac{y \cos xy - 1}{1 - x \cos xy}$
 (C) $\frac{\cos xy - 1}{1 - x \cos xy}$
 (D) $\frac{1}{\cos xy - 1}$
 (E) $\frac{\cos x - 1}{1 - \cos y}$

The graph of $f(x)$ consists of four line segments as shown below. Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$. Use this information for Problems 26–28.



26. What is $g(0)$?

- (A) 4
- (B) 2
- (C) 0
- (D) -2
- (E) -4

27. What is the equation of the tangent to $g(x)$ at the point $[3, g(3)]$?

- (A) $y = 0$
- (B) $y = 1$
- (C) $y = x - 3$
- (D) $y = x + 3$
- (E) $y = -3$

28. Which of the following is false for $g(x)$?

- (A) $g(x)$ has a relative maximum at $x = -1$.
- (B) $g(x)$ has a relative minimum at $x = 3$.
- (C) $g(x)$ has a relative maximum at $x = -2$.
- (D) $g(x)$ is decreasing in the interval $2 < x < 3$.
- (E) $g(x)$ is increasing in the interval $-2 < x < -1$.

Section I, Part B: Multiple-Choice Questions

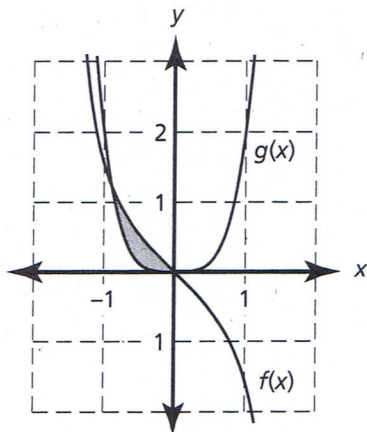
Time: 50 minutes

Number of Questions: 17

A calculator may be used on this part of the examination.

29) What is the area of the first closed region to the left of the y -axis, bounded by the curves $f(x) = -\tan x$ and $g(x) = 2x^4$?

- (A) 1.206
- (B) 0.931
- (C) 0.691
- (D) 0.452
- (E) 0.240



30. What is the average rate of change

of $f(x) = \frac{e^x}{x^2}$ in the interval

$-4 \leq x \leq -1$?

- (A) 0.106
- (B) 0.137
- (C) 0.319
- (D) 0.411
- (E) 1.233

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31. Consider the integral expression

$$\int_0^{\frac{\pi}{2}} \sin(2x)e^{\cos(2x)} dx. \text{ If } u = \cos 2x$$

then which integral below is equivalent to the given integral?

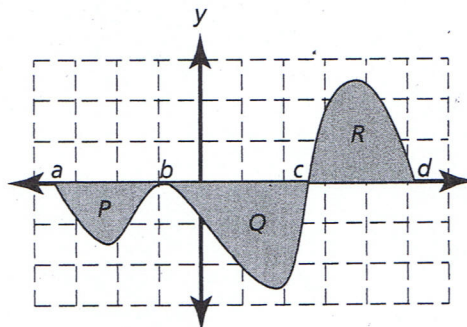
- (A) $-\frac{1}{2} \int_0^{\pi} e^u du$
 (B) $-2 \int_0^{\pi} e^u du$
 (C) $-\frac{1}{2} \int_{-1}^1 e^u du$
 (D) $\frac{1}{2} \int_{-1}^1 e^u du$
 (E) $2 \int_{-1}^1 e^u du$
32. Let $f(x) = \frac{1}{x}$ and $k > 1$. If the area between the x -axis and the graph of $f(x)$ in the closed interval $k \leq x \leq k+1$ is 0.125 where $k > 1$, then what is the value of k ?
- (A) 0.133
 (B) 1.133
 (C) 1.334
 (D) 2.998
 (E) 7.510
33. A solid has its base in the xy -plane, bounded by the x -axis, the y -axis, and the function $y = 3 - x^5$. If cross sections taken perpendicular to the x -axis are semicircles whose diameters are in the xy -plane, what is the volume of this solid?
- (A) 3.335
 (B) 4.247
 (C) 5.239
 (D) 6.671
 (E) 13.342

34. Shampoo drips from a crack in the side of a plastic bottle at a rate

$$\text{modeled by } Y(t) = \frac{t}{\sqrt{1+t^{\frac{3}{2}}}}, \text{ where}$$

$Y(t)$ is in ounces per minute. If there are 32 ounces in the bottle at $t = 0$, how many ounces are left in the bottle after 5 minutes?

- (A) 26.937 ounces
 (B) 24.355 ounces
 (C) 7.645 ounces
 (D) 5.063 ounces
 (E) The bottle will be empty before 5 minutes has elapsed.
35. Consider the function $f(x) = x^3 + 2$ in the closed interval $0 < a \leq c \leq 2$. If the value guaranteed by the Mean Value Theorem in the closed interval is $c = 1.720$, then what is the value of a ?
- (A) 1.260
 (B) 1.424
 (C) 1.602
 (D) 1.680
 (E) none of these
36. The sketch of $f(x)$ is shown below, with regions bounded by $f(x)$ and the x -axis indicated by P , Q , and R . If $\int_a^d f(x) dx = -7$, $\int_b^d f(x) dx = -2$ and $\int_c^a f(x) dx = 17$, what is $\int_b^c f(x) dx$?



- (A) -12
 (B) -6
 (C) -3
 (D) 4
 (E) none of these

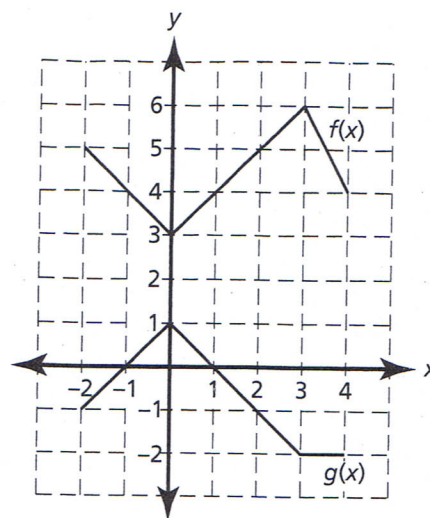
37. Let $h(x) = xg(x)$, where $g(x) = f^{-1}(x)$. Use the table of values below to find $h'(5)$.

x	$f(x)$	$f'(x)$
2	4	-1
3	5	2
5	1	3

- (A) $\frac{1}{2}$
 (B) 2.5
 (C) 3
 (D) $4\frac{2}{3}$
 (E) 5.5
38. Let $f(x) = \sin x$ and $g(x) = p \ln x$ in the closed interval $0 \leq x \leq \frac{\pi}{2}$. For what value of p will the tangents to the curves at their points of intersection be perpendicular?
- (A) -0.447
 (B) 0.410
 (C) 1.260
 (D) 1.303
 (E) none of these
39. The height of a conical sand pile is always twice the radius. If sand is being added to the pile at a rate of 30π cm³/min, how fast is the height of the pile increasing when the circumference of the base of the sand pile is 120π cm?

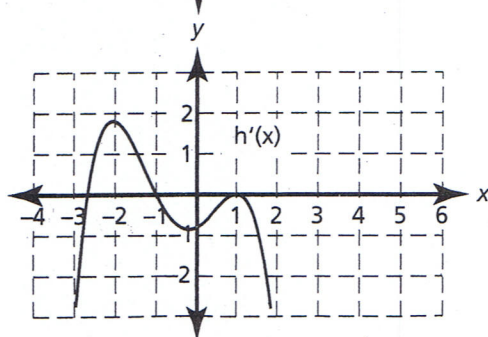
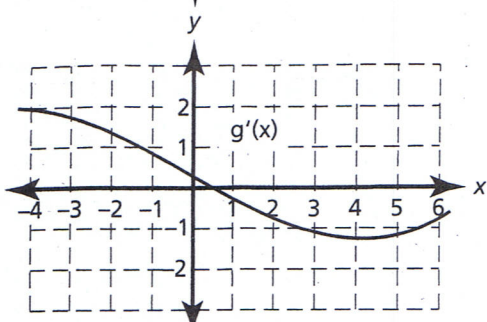
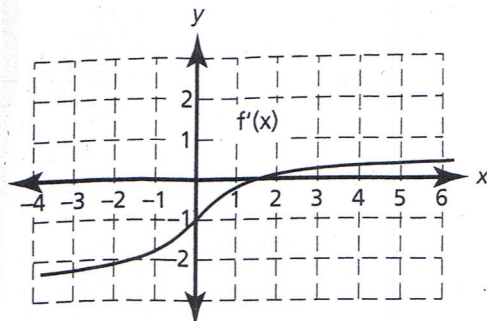
$$(V_{\text{cone}} = \frac{\pi}{3} r^2 h)$$

- (A) $\frac{1}{120\pi}$ cm/min
 (B) $\frac{1}{120}$ cm/min
 (C) $\frac{2}{15}$ cm/min
 (D) $\frac{1}{4}$ cm/min
 (E) none of these



40. The graphs of $f(x)$ and $g(x)$ are shown above. If $h(x) = \frac{g(2x)}{f(x)}$, use the graphs to find $h'(1)$.
- (A) $-\frac{7}{4}$
 (B) $-\frac{9}{16}$
 (C) $-\frac{7}{16}$
 (D) $-\frac{5}{16}$
 (E) $-\frac{3}{16}$
41. The number of home fires each day in a certain city increases as the temperature drops. The rate of home fires is modeled by
- $$F(t) = 4 \cos\left(\frac{t}{58} - 2\right) + 6, \text{ for}$$
- $0 \leq t \leq 365$ days, where midnight on January 1st corresponds to $t = 0$. Which of the following is closest to the approximate number of fires in the first quarter of the year?
- (A) 910
 (B) 660
 (C) 540
 (D) 330
 (E) 240

42. The graphs of the derivatives of three functions, f , g , and h , are given below. Which of the functions has at least one point of inflection in the open interval $-3 < x < 2$?



- (A) only $f(x)$
 (B) only $g(x)$
 (C) only $h(x)$
 (D) only $f(x)$ and $g(x)$
 (E) only $g(x)$ and $h(x)$

x	f	g	f'	g'
1	3	4	$\frac{2}{3}$	$-\frac{5}{2}$
2	4	2	$\frac{4}{3}$	$-\frac{3}{2}$
4	8	1	$\frac{8}{3}$	$\frac{1}{2}$

43. If $f(x)$ and $g(x)$ are differentiable functions with values as given in the chart above, and $k(x) = f(g(x^2))$, what is $k'(2)$?

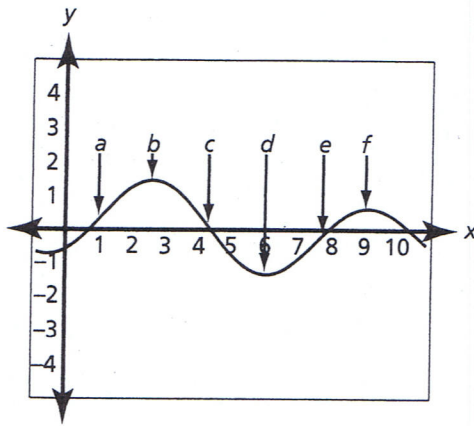
- (A) $\frac{1}{3}$
 (B) $\frac{2}{3}$
 (C) $\frac{4}{3}$
 (D) $\frac{16}{3}$
 (E) none of these

44. The price of a newly issued stock varies sinusoidally during the first 10 days after its initial offering and is modeled by

$$P(t) = \log(2t + 1) \sin t + 20,$$

where t is in days. To the nearest cent, what is the price of the stock when the price of the stock is decreasing most rapidly in the interval $0 \leq t \leq 10$?

- (A) \$7.98
 (B) \$9.49
 (C) \$19.91
 (D) \$20.12
 (E) \$21.22



45. The graph of $g'(x)$ is shown on the graph to the left. For which of the stated interval(s) is the function $g(x)$ both increasing and concave up?
- (A) $a < x < b$
 (B) $e < x < f$
 (C) $a < x < b$ and $e < x < f$
 (D) $a < x < c$ and $e < x < h$
 (E) $a < x < b$ and $d < x < f$

Section II
Free-Response Questions
Time: 1 hour and 30 minutes
Number of Problems: 6

Part A
Time: 45 minutes
Number of Problems: 3

You may use a calculator for any problem in this section.

- The velocity of a particle moving along the x -axis is given by $v(t) = \frac{e^{t-1}}{2t^2 + 1} - t^2$ for $0 \leq t \leq 12$. The position of the particle $x(t)$ is 3 when t is 2.
 - During what interval is the particle moving to the left? Explain your reasoning.
 - What is the position of the particle when it is farthest to the left?
 - At what time in the interval $5 \leq t \leq 10$ is the instantaneous velocity equal to the average velocity?
 - How far did the particle travel on $0 \leq t \leq 12$?
- A solution is draining through a conical filter into an identical conical container as shown in the diagram to the right. The solution drips from the upper filter into the lower container at a rate of $\pi \text{ cm}^3/\text{sec}$ ($V_{\text{cone}} = \frac{\pi}{3} r^2 h$).
 - How fast is the level in the upper filter dropping when the solution level in the upper filter is at 6 cm?
 - If the conical filter is initially full, what is the level of the solution in the lower level when the solution level in the upper filter is at 6 cm and how fast is the level in the lower filter rising?
 - How fast is the surface area of the solution in the lower filter increasing when the volume in the upper filter equals the volume in the lower container?

