Key

Calculus BC – Unit 3 – Taylor and McLaurin Series Five' N One- Calculator Inactive - Afternoon

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D

What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1<x<1$

(B) $x>1$ only

(C) $x\geq 1$ only

(D) $x<-1$ and $x>1$ only

(E) $x\leq -1$ and $x\geq 1$

geo

$\frac{2}{x^2+1}<1$

$2<x^2+1$

$x^2>1$

$x>1$ or $x<-1$

D

Consider the series $\sum_{n=1}^{\infty}\frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty}\frac{e}{n!}<1$

(B) $\lim_{n\to\infty}\frac{n!}{e}<1$

(C) $\lim_{n\to\infty}\frac{n+1}{e}<1$

(D) $\lim_{n\to\infty}\frac{e}{n+1}<1$

(E) $\lim_{n\to\infty}\frac{e}{(n+1)!}<1$

$\lim_{n\to\infty}\left|\frac{e^{n+1}}{(n+1)!}\cdot\frac{n!}{e^n}\right|$ $\quad \frac{\cancel{e^n}\cdot e}{(n+1)\cdot \cancel{n!}}\cdot\frac{\cancel{n!}}{\cancel{e^n}}$

$\lim_{n\to\infty}\left|\underbrace{\frac{1}{n+1}}_{=0}\cdot e\right|$

E

If $f(x)=x\sin(2x)$, which of the following is the Taylor series for f about $x=0$?

(A) $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$

(B) $x-\frac{4x^3}{2!}+\frac{16x^5}{4!}-\frac{64x^7}{6!}+\cdots$

(C) $2x-\frac{8x^3}{3!}+\frac{32x^5}{5!}-\frac{128x^7}{7!}+\cdots$

(D) $2x^2-\frac{2x^4}{3!}+\frac{2x^6}{5!}-\frac{2x^8}{7!}+\cdots$

(E) $2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}-\frac{128x^8}{7!}+\cdots$

$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}$

$\sin(2x)= 2x-\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}$

$x\sin(2x)= 2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}$

D

Which of the following series converges for all real numbers x?

(A) $\sum_{n=1}^{\infty}\frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty}\frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty}\frac{n!x^n}{e^n}$

$\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}$ $\quad 1\cdot|x|<1$

$\frac{x^{n+1}}{n+1}\cdot\frac{n^2}{x^n}$ $\quad 1\cdot|x|<1$

$\frac{x^{n+1}}{(n+1)^{1/2}}\cdot\frac{n^{1/2}}{x^n}$ $\quad 1\cdot|x|<1$

$\frac{e^{n+1}\cdot x^{n+1}}{(n+1)!}\cdot\frac{n!}{e^n x^n}$

$\frac{e\cdot x}{n+1}$

$0\cdot|x|<1$

C

What is the sum of the series $1+\ln 2+\frac{(\ln 2)^2}{2!}+\cdots+\frac{(\ln 2)^n}{n!}+\cdots$?

(A) $\ln 2$

(B) $\ln(1+\ln 2)$

(C) 2

(D) e^2

(E) The series diverges.

$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$

e^x

$e^{\ln 2}=2$

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Calculator Active

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Let h be a function having derivatives of all orders for x > 0. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3$

a) Write the first-degree Taylor polynomial for h about x=2 and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

$P_1(x) = 80 + 128(x-2)$

$h(1.9) \approx P_1(1.9) = 80 + 128(1.9-2)$

$P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ because h' is inc $\therefore$ cc↑ tan line $\therefore$ approximation is below

b) Write the third-degree Taylor polynomial for h about x=2 and use it to approximate $h(1.9)$.

$P_3(x) = 80 + 128(x-2) + \frac{488}{3} \cdot \frac{(x-2)^2}{2!} + \frac{448}{3} \cdot \frac{(x-2)^3}{3!}$

$h(1.9) \approx P_3(1.9) =$

$80 + 128(1.9-2) + \frac{488}{3} \cdot \frac{(1.9-2)^2}{2!} + \frac{448}{3} \cdot \frac{(1.9-2)^3}{3!}$

c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about x = 2 approximates $h(1.9)$ with error less than $3x10^{-4}$.

max $h^4(x) = \frac{584}{9}$

$1.9 \le z \le 2$

4th der is inc. on interval so max at $\frac{584}{9}$

$|h(1.9) - P_3(1.9)| \le \left| \frac{\frac{584}{9}(1.9-2)^4}{4!} \right|$

2.7037×10^{-4}

$< 3 \times 10^{-4}$

(a) $P_1(x) = 80 + 128(x-2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \le x \le 3$.

$4 : \begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$

(b) $P_3(x) = 80 + 128(x-2) + \frac{488}{6}(x-2)^2 + \frac{448}{18}(x-2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

$3 : \begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$

(c) The fourth derivative of h is increasing on the interval $1 \le x \le 3$, so $\max\limits_{1.9 \le x \le 2} \left|h^{(4)}(x)\right| = \frac{584}{9}$.

Therefore, $\left|h(1.9) - P_3(1.9)\right| \le \frac{584}{9} \frac{|1.9-2|^4}{4!}$
$= 2.7037 \times 10^{-4}$
$< 3 \times 10^{-4}$

$2 : \begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$