Practice Exam:

1) $f(x) = \dfrac{x^2}{\tan 2x}$

B

$$f'(x) = \frac{(\tan 2x)(2x) - x^2 \cdot \sec^2 2x \cdot 2}{(\tan 2x)^2}$$

2) $x(t) = t^3 - 6t^2 \quad y(t) = t^2 - 16$

B

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2 - 12t}$$

$3t^2 - 12t = 0$
$3t(t-4) = 0$
$t = \cancel{0}, 4$

3) $\displaystyle\int_2^\infty \frac{\cos(2x^{-1})}{x^2}\,dx = \lim_{b\to\infty} \int_2^b \frac{\cos(2x^{-1})}{x^2}\,dx$

C

$u = 2x^{-1}$
$du = -2x^{-2}dx$
$-\frac{1}{2}du = \frac{1}{x^2}dx$

$-\frac{1}{2}\int \cos u\,du$
$-\frac{1}{2}\sin(2x^{-1})$

$$\lim_{b\to\infty} \frac{-1}{2}\sin\left(\frac{2}{x}\right)\Big|_2^b = \lim_{b\to\infty} \frac{-1}{2}\left(\sin\frac{2}{b} - \sin 1\right)$$
$$= \frac{-1}{2}(-\sin 1) = \frac{1}{2}\sin 1$$

4) $\sin 2x + \cos 2y = x - y$

A

$$2\cos 2x - 2\sin 2y\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\frac{dy}{dx}(1 - 2\sin 2y) = 1 - 2\cos 2x$$
$$\frac{dy}{dx} = \frac{1 - 2\cos 2x}{1 - 2\sin 2y}$$

5) $\displaystyle\lim_{\omega\to 0} \frac{\ln\left(\frac{2+\omega}{2}\right)}{\omega}$ $\quad \frac{0}{0} \therefore$ L'Hopital $\quad \frac{2+\omega}{2} = 1 + \frac{1}{2}\omega$

B

$$\lim_{\omega\to 0}\left(\frac{2}{2+\omega}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$$

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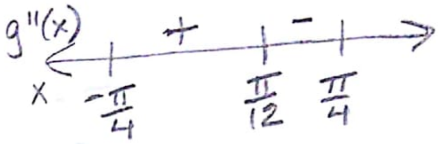
6) $g(x) = \sin 2x + x^2 \quad -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

C $\quad g'(x) = 2\cos 2x + 2x$

$g''(x) = -4\sin 2x + 2 = 0$

$-4\sin 2x = -2$

$\sin 2x = \frac{1}{2}$



$2x = \frac{\pi}{6}$

$x = \frac{\pi}{12}$

7) II. not diff at $x = 1.5$

A $\quad$ III. not cont. at $x = 1$

$\therefore$ I only

8) slope field

E

9) $F'(x) = \sec^2(3x)$

C $\quad F(x) = \int \sec^2 3x \, dx \qquad u = 3x$

$du = 3dx$

$\frac{1}{3}\tan 3x + C \qquad \frac{1}{3}du = dx$

10) $2 \cdot \int_0^{\pi/3} (2\cos x - 1)dx = 2(2\sin x - x) \Big|_0^{\pi/3} \qquad 2\cos x = 1$

D $\qquad \cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

$= 2\left[\left(2\sin\frac{\pi}{3} - \frac{\pi}{3}\right) - (0)\right]$

$2\left[2 \cdot \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$

$2\sqrt{3} - \frac{2\pi}{3}$

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) $y = -\sin x$

E $y' = -\cos x$

$y'\left(\frac{2\pi}{3}\right) = -\cos\frac{2\pi}{3} = \frac{1}{2}$

$y\left(\frac{2\pi}{3}\right) = -\sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

$y + \frac{\sqrt{3}}{2} = \frac{1}{2}\left(x - \frac{2\pi}{3}\right)$

12)

A

$$\frac{\int_{\pi/4}^{\pi/3} (\tan x - \sin x)\,dx}{\frac{\pi}{3} - \frac{\pi}{4}}$$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ $\quad u = \cos x$, $du = -\sin x\,dx$, $-du = \sin x\,dx$

$-\int \frac{1}{u}\,du = -\ln|\cos x|$

$$\frac{-\ln|\cos x| + \cos x \Big|_{\pi/4}^{\pi/3}}{\frac{\pi}{12}} = \frac{\left(-\ln\left|\cos\frac{\pi}{3}\right| + \cos\frac{\pi}{3}\right) - \left(-\ln\left|\cos\frac{\pi}{4}\right| + \cos\frac{\pi}{4}\right)}{\frac{\pi}{12}}$$

$\frac{12}{\pi}\left(-\ln\frac{1}{2} + \frac{1}{2} + \ln\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$

$\frac{12}{\pi}\left(\ln\frac{\sqrt{2}}{2}\cdot 2 - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) = \frac{12}{\pi}\left(\ln\sqrt{2} + \frac{1-\sqrt{2}}{2}\right)$

13)

D

$f'(x) = -x^4$

$g'(x) = \sin(\pi x)$

$h'(x) = 3 - 2x$

$f'\left(\frac{3}{2}\right) = -$

$g'\left(\frac{3}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

$h'\left(\frac{3}{2}\right) = 3 - 2\left(\frac{3}{2}\right) = 0$

$\sin(\pi x) = 0$

$\pi x = 0$ $\quad \pi x = \pi$

$x = 0$ $\quad x = 1$

$\pi x = 2\pi$

$x = 2$

14) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

B

$e^{-\pi^2} = 1 - \pi^2 + \frac{\pi^4}{2!} - \frac{\pi^6}{3!}$

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15) $\frac{dy}{dx} = -\frac{1}{2}x$

E

$\int dy = \int -\frac{1}{2}x\,dx$

$y = -\frac{1}{2}\int x\,dx$

$y = -\frac{1}{2}\cdot\frac{1}{2}x^2 + C$

$y = -\frac{1}{4}x^2 + C$

$0 = 0 + C$

$y = -\frac{1}{4}x^2$

16) $f(x) = \tan x$

A

$f'(x) = \sec^2 x = (\sec x)^2$

$f''(x) = 2\cdot\sec x\cdot\sec x\tan x$

$f''\left(\frac{\pi}{3}\right) = 2\cdot 2\cdot 2\cdot\sqrt{3} = 8\sqrt{3}$

17) $A = \frac{1}{2}\int_0^{\pi}\left[\theta^2 - \underbrace{\left(\frac{1}{3}\theta\right)^2}_{\frac{1}{9}\theta^2}\right]d\theta = \frac{1}{2}\left[\frac{\theta^3}{3} - \frac{\theta^3}{27}\Big|_0^{\pi}\right]$

C

$\frac{1}{2}\left[\frac{\pi^3}{3} - \frac{\pi^3}{27}\right] = \frac{1}{2}\left[\frac{9\pi^3}{27} - \frac{\pi^3}{27}\right] = \frac{4\pi^3}{27}$

18)

D

x	y	dy/dx
0	3	−2
0.5	2	−3/2
1		

$y = -2(0.5) + 3 = 2$

$y = -\frac{3}{2}\left(\frac{1}{2}\right) + 2 = -\frac{3}{4} + \frac{8}{4} = \frac{5}{4}$

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19) $\frac{dy}{dx} = \frac{2x+1}{3y^2}$ $y=2$ $x=1$ $y=?$ $x=3$

E

$\int 3y^2 dy = \int (2x+1) dx$

$y^3 = x^2 + x + C$

$8 = 1 + 1 + C$

$C = 6$

$y^3 = x^2 + x + 6$

$y^3 = 9 + 3 + 6$

$y^3 = 18$

$y = \sqrt[3]{18}$

20)

E $\lim_{x\to 4^-} f(x) = \ln 4 - \frac{4}{4} = \ln 4 - 1$

$\lim_{x\to 4^+} f(x) = \ln 4 - \frac{4^2}{16} = \ln 4 - 1$ $\therefore$ cont.

$f(4) = \ln 4 - 1$

$\lim_{x\to 4^-} f'(x) = \frac{-1}{4}$ $\therefore$ diff

$\lim_{x\to 4^+} f'(x) = \left(\frac{1}{x} - \frac{1}{8}x\right) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ ✓

21) $\int \frac{1}{5x - x^2} dx$ $\frac{1}{x(5-x)} = \frac{A}{x} + \frac{B}{5-x}$

D

$x(5-x)$ $1 = A(5-x) + Bx$

$x=5:\ 1 = 5B \quad B = \frac{1}{5}$

$\frac{1}{5}\int \frac{1}{x} dx + \frac{1}{5}\int \frac{1}{5-x} dx$ $x=0:\ 1 = 5A \quad A = \frac{1}{5}$

$\frac{1}{5}\ln|x| - \frac{1}{5}\ln|5-x| + C$

$\frac{1}{5}\ln\left|\frac{x}{5-x}\right| + C$

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22) $g(x) = 6 + \int_{-2}^{x} \cos(\omega^5)\,d\omega$

C

$g'(x) = \cos(x^5)$

23) $2\int \arctan x\,dx$ ILATE

B

$u = \arctan x \quad v = x$

$du = \frac{1}{1+x^2}dx \quad dv = dx$

$2\left(x\cdot\arctan x - \int \frac{x}{1+x^2}dx\right)$ $\quad u = 1+x^2 \quad \frac{1}{2}\int \frac{1}{u}du$

$du = 2x\,dx$

$2x\arctan x - \ln(1+x^2) + C \quad \frac{1}{2}du = x\,dx \quad \frac{1}{2}\ln(1+x^2)$

24) $g(x) = \int_0^x f(t)\,dt - \int_2^x f(t)\,dt$

A

$g'(x) = f(x) - f(x) = 0$

25) $\lim_{x\to\infty} \frac{\arctan x + 2x}{x + e^{-x}} \quad \frac{\infty}{\infty}$ $\therefore$ L'Hopital

D

$\lim_{x\to\infty} \frac{\frac{1}{1+x^2} + 2}{1 - e^{-x}} = \frac{2}{1} = 2$

26) $y = \sin x \quad \sin\pi = 0$

A

$y' = \cos x \quad f'(\pi) = -1$

$y'' = -\sin x \quad f''(\pi) = 0$

$y''' = -\cos x \quad f'''(\pi) = 1$

$P(x) = 0 + -1(x-\pi) + \frac{(x-\pi)^3}{3!}$

$-1(3-\pi) + \frac{(3-\pi)^3}{3!}$

D 27) $s(t) = \langle 2\sin 4t, -3\cos 4t\rangle$

$v(t) = \langle 8\cos 4t, 12\sin 4t\rangle$

$a(t) = \langle -32\sin 4t, 48\cos 4t\rangle$

$a(\frac{\pi}{4}) = \langle 0, -48\rangle$

$\sqrt{0^2 + (-48)^2} = 48$

28)

E