1. $\int (3x^2 - 2x + 3)\,dx =$

(A) $x^3 - x^2 + C$ (B) $3x^3 - x^2 + 3x + C$ (C) $x^3 - x^2 + 3x + C$

(D) $\frac{1}{2}(3x^2 - 2x + 3)^2 + C$ (E) none of these

20. $\int \frac{x^3 - x - 1}{x^2}\,dx =$

(A) $\dfrac{\frac{1}{4}x^4 - \frac{1}{2}x^2 - x}{\frac{1}{3}x^3} + C$

(B) $1 + \frac{1}{x^2} + \frac{2}{x^3} + C$

(C) $\frac{x^2}{2} - \ln|x| - \frac{1}{x} + C$

(D) $\frac{x^2}{2} - \ln|x| + \frac{1}{x} + C$

(E) $\frac{x^2}{2} - \ln|x| + \frac{2}{x^3} + C$

3. Let $f(x)$ be defined as below. Evaluate $\int_0^6 f(x)\,dx$.

$$f(x) = \begin{cases} x & 0 < x \leq 2 \\ 1 & 2 < x \leq 4 \\ \frac{1}{2}x & 4 < x \leq 6 \end{cases}$$

A) 5

B) 6

C) 7

D) 8

E) 9

77. The equation of the curve whose slope at point (x, y) is $x^2 - 2$ and which contains the point $(1, -3)$ is

(A) $y = \frac{1}{3}x^3 - 2x$ (B) $y = 2x - 1$ (C) $y = \frac{1}{3}x^3 - \frac{10}{3}$

(D) $y = \frac{1}{3}x^3 - 2x - \frac{4}{3}$ (E) $3y = x^3 - 10$

8. Which of the following statements are true?

I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand Riemann sum.

II. If the function f is continuous on the interval $[a, b]$ and $\int_a^b f(x)\,dx = 0$, then f must have at least one zero between a and b.

III. If $f'(x) > 0$ for all x in an interval, then the function f is concave up in that interval.

A) I only

B) II only

C) III only

D) II and III only

E) None are true.

1101 (1999AB, Calculator). The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	0	3	6	9	12	15	18	21	24
$R(t)$ (gal/hr)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

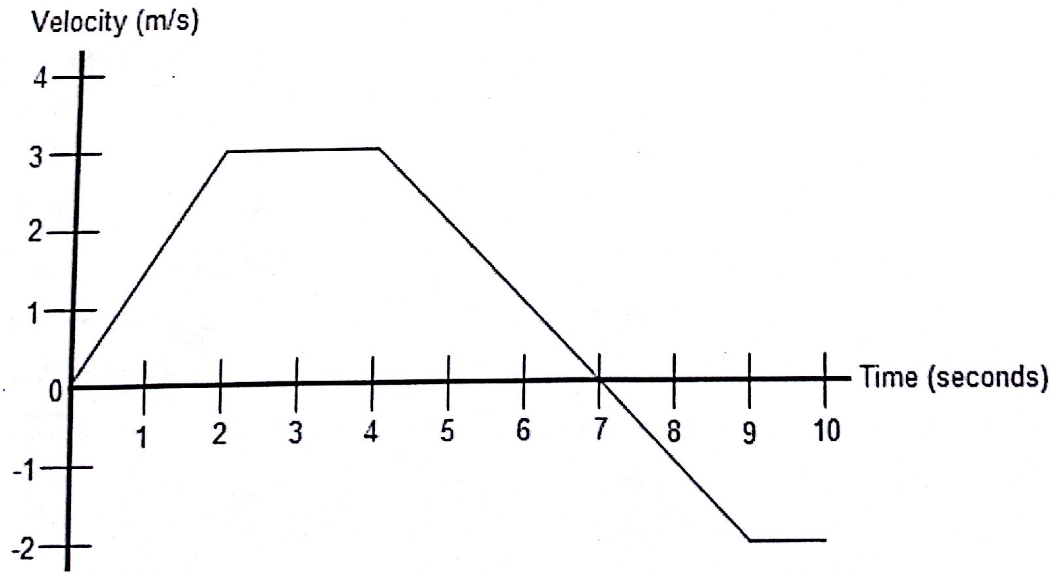
b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

922. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table.

Time (hours)	0	1	2	3	4	5	6	7	8
Leakage (gal./hour)	50	70	97	136	190	265	369	516	720

a) Give an upper and lower estimate of the total quantity of oil that has escaped after 5 hours.

An object's velocity during a 10 second time interval is shown by the graph below:



a.) Determine the object's total distance traveled and displacement.
b.) At $t = 0$, the object's position is $x = 2$ m. Find the object's position at $t = 2$, $t = 4$.

5 Key

1. $\int(3x^2-2x+3)\,dx=$

C

(A) x^3-x^2+C (B) $3x^3-x^2+3x+C$ (C) x^3-x^2+3x+C

(D) $\frac{1}{2}(3x^2-2x+3)^2+C$ (E) none of these

$\frac{\cancel{3}x^3}{\cancel{3}}-\frac{\cancel{2}x^2}{\cancel{2}}+3x+C$

20. $\int\frac{x^3-x-1}{x^2}\,dx=$

D

(A) $\dfrac{\frac{1}{4}x^4-\frac{1}{2}x^2-x}{\frac{1}{3}x^3}+C$

(B) $1+\frac{1}{x^2}+\frac{2}{x^3}+C$

(C) $\frac{x^2}{2}-\ln|x|-\frac{1}{x}+C$

(D) $\frac{x^2}{2}-\ln|x|+\frac{1}{x}+C$

(E) $\frac{x^2}{2}-\ln|x|+\frac{2}{x^3}+C$

$x-\frac{1}{x}-x^{-2}$

$\frac{x^2}{2}-\ln|x|-\frac{x^{-1}}{-1}$

$\frac{x^2}{2}-\ln|x|+\frac{1}{x}+C$

3. Let $f(x)$ be defined as below. Evaluate $\int_0^6 f(x)\,dx$.

E

$$f(x)=\begin{cases}x & 0<x\le 2\\ 1 & 2<x\le 4\\ \frac{1}{2}x & 4<x\le 6\end{cases}$$

$\frac{x^2}{2}\Big|_0^2$ $2-0=2$

$x\Big|_2^4$ 2

$\frac{1}{4}x^2\Big|_4^6$ $9-4=5$

A) 5

B) 6

C) 7

D) 8

E) 9

77. The equation of the curve whose slope at point (x, y) is x^2-2 and which contains the point $(1, -3)$ is

D

(A) $y=\frac{1}{3}x^3-2x$ (B) $y=2x-1$ (C) $y=\frac{1}{3}x^3-\frac{10}{3}$

(D) $y=\frac{1}{3}x^3-2x-\frac{4}{3}$ (E) $3y=x^3-10$

$\int(x^2-2)\,dx$

$\frac{x^3}{3}-2x+C$

$-3=\frac{1}{3}-2(1)+C$

$-3=\frac{1}{3}-2+C$

$-3=-\frac{5}{3}+C$

$-\frac{9}{3}=-\frac{5}{3}+C$

$C=-\frac{4}{3}$

$3\left(\frac{x^3}{3}-2x+\frac{-4}{3}\right)$

x^3-6x-4

8. Which of the following statements are true?

F I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand Riemann sum.

II. If the function f is continuous on the interval $[a,b]$ and $\int_a^b f(x)\,dx=0$, then f must have at least one zero between a and b.

F III. If $f'(x)>0$ for all x in an interval, then the function f is concave up in that interval.

B

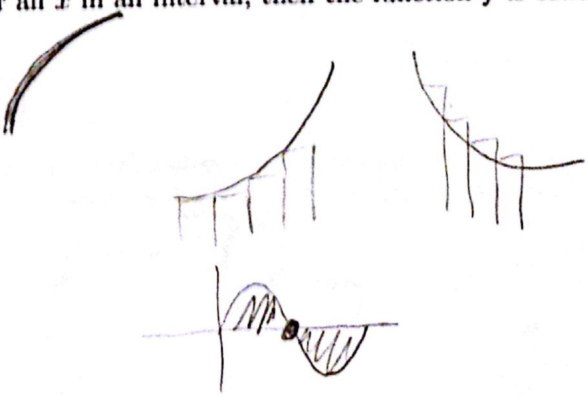
A) I only

B) II only

C) III only

D) II and III only

E) None are true.



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1101 (1999AB, Calculator). The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	0	3	6	9	12	15	18	21	24
$R(t)$ (gal/hr)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

(2) a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(2) b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

a) $6(10.4) + 6(11.2) + 6(11.3) + 6(10.2) = 258.6$ gal

Between 0 and 24 hrs, about 258.6 gal flow out

b) yes, $R(0) = R(24)$, RT guarantees there is a t, $0<t<24$ such that $R'(t) = 0$

922. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table.

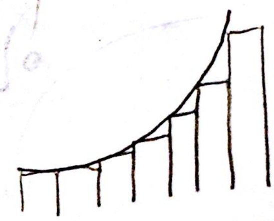
Time (hours)	0	1	2	3	4	5	6	7	8
Leakage (gal./hour)	50	70	97	136	190	265	369	516	720

a) Give an upper and lower estimate of the total quantity of oil that has escaped after 5 hours.

~~b)~~ Give an upper and lower estimate of the total quantity of oil that has escaped after 8 hours.

(2) LRAM: $1(50+70+97+136+190) = 543$ gal lower

RRAM: $1(70+97+136+190+265) = 758$ gal upper



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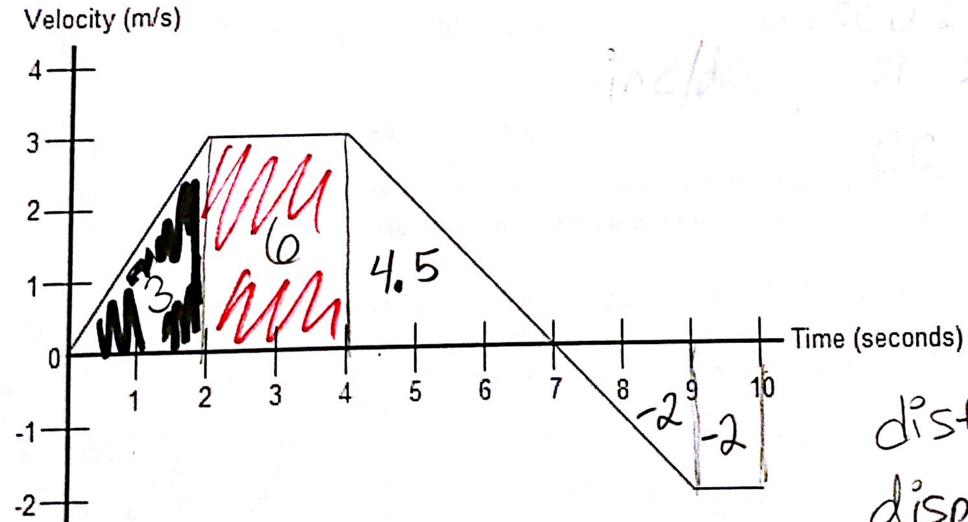

dist = 17.5 m (1)

displace = 9.5 m (1)

a.) Determine the object's total distance traveled and displacement.

b.) At $t = 0$, the object's position is $x = 2$ m. Find the object's position at $t = 2$, $t = 4$.

$t=2$: $2 + \underline{3} = 5$ m (1)

$t=4$: $2 + 9 = 11$ m (1)