Polar Quiz Review:

1) $r\sin\theta = 4$ ($y = 4$)

$r = 4\csc\theta$

2) $3x - 5y + 2 = 0$

$3r\cos\theta - 5r\sin\theta + 2 = 0$

$r = \dfrac{-2}{3\cos\theta - 5\sin\theta}$

3) $x^2 + y^2 = 25$

$r^2 = 25$

$r = 5$

4) $r = 3\sec\theta$

$r = \dfrac{3}{\cos\theta}$

$r\cos\theta = 3$

$x = 3$

5) $r = 2\sin\theta$

$\dfrac{r}{2} = \sin\theta$

$\dfrac{r}{2} = \dfrac{y}{r}$

$r^2 = 2y$

$x^2 + y^2 = 2y$

6) $\theta = \dfrac{5\pi}{6}$

$\tan\theta = \dfrac{y}{x}$

$\tan\dfrac{5\pi}{6} = \dfrac{y}{x}$

$\dfrac{y}{x} = -\dfrac{\sqrt{3}}{3}$

$y = \dfrac{-x}{\sqrt{3}}$

Quiz Review:

7) $r = 2 + 3\sin\theta$ $\quad \sin\frac{3\pi}{2} = -1$

$r' = 3\cos\theta$ $\quad \cos\frac{3\pi}{2} = 0$

$$\frac{dy}{dx} = \frac{(2+3\sin\theta)\cos\theta + 3\cos\theta\sin\theta}{(-2-3\sin\theta)\sin\theta + 3\cos\theta\cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{2}} = \frac{(2+3\cdot-1)(0)+(3\cdot0\cdot-1)}{(-2-3\cdot-1)(-1)+3(0)(0)} = \boxed{0}$$

8) $r = 3(1-\cos\theta)$ $\quad \theta = \frac{\pi}{2}$

$r = 3 - 3\cos\theta$ $\quad \sin\frac{\pi}{2} = 1$

$r' = 3\sin\theta$ $\quad \cos\frac{\pi}{2} = 0$

$$\frac{dy}{dx} = \frac{(3-3\cos\theta)(\cos\theta) + (3\sin\theta)(\sin\theta)}{(-3+3\cos\theta)(\sin\theta) + (3\sin\theta)(\cos\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{(3-3\cdot0)(0)+(3\cdot1)(1)}{(-3+3\cdot0)(1)+(3\cdot1)(0)} = \frac{3}{-3} = \boxed{-1}$$

9) $r = 4\sin\theta$ $\quad \theta = \frac{\pi}{3}$

$r' = 4\cos\theta$ $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{(4\sin\theta)(\cos\theta) + (4\cos\theta)(\sin\theta)}{(-4\sin\theta)(\sin\theta) + (4\cos\theta)(\cos\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{(4\cdot\frac{\sqrt{3}}{2})(\frac{1}{2}) + (4\cdot\frac{1}{2})(\frac{\sqrt{3}}{2})}{(-4\cdot\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2}) + (4\cdot\frac{1}{2})(\frac{1}{2})} = \frac{\sqrt{3}+\sqrt{3}}{-3+1}$$

$$\frac{2\sqrt{3}}{-2} = \boxed{-\sqrt{3}}$$

10) $r = 2\sin(3\theta)$, $\theta = \frac{\pi}{4}$

$r' = 6\cos(3\theta)$ $\quad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$\frac{dy}{dx} = \frac{2\sin(3\theta)\cdot\cos\theta + 6\cos(3\theta)\cdot\sin\theta}{-2\sin(3\theta)\cdot\sin\theta + 6\cos(3\theta)\cdot\cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 6\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{-2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 6\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{1 + -3}{-1 - 3} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$$

11) $r = 1 + \sin\theta$ $\quad r' = \cos\theta$

H: $(1+\sin\theta)(\cos\theta) + \cos\theta\sin\theta = 0$

$\cos\theta + \sin\theta\cos\theta + \sin\theta\cos\theta = 0$

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1 + 2\sin\theta) = 0$

$\cos\theta = 0$ $\quad 2\sin\theta = -1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sin\theta = -\frac{1}{2}$

$r = 2$ $\quad r = 0$ $\quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$r = \frac{1}{2}$ $\quad r = \frac{1}{2}$

$(2, \frac{\pi}{2})$

~~$(0, \frac{3\pi}{2})$~~

$(\frac{1}{2}, \frac{7\pi}{6})$

$(\frac{1}{2}, \frac{11\pi}{6})$

V: $(-1-\sin\theta)(\sin\theta) + \cos\theta\cos\theta = 0$

$-\sin\theta - \sin^2\theta + \cos^2\theta = 0$

$-\sin\theta - \sin^2\theta + (1-\sin^2\theta) = 0$

$-\sin\theta - \sin^2\theta + 1 - \sin^2\theta = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

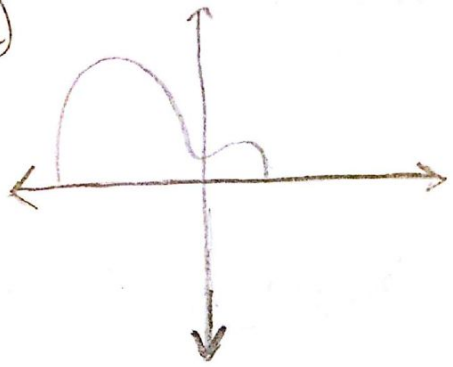
$\sin\theta = -1$ $\quad \theta = \frac{3\pi}{2}$

$(\frac{3}{2}, \frac{\pi}{6})$

$(\frac{3}{2}, \frac{5\pi}{6})$

12) $r = \theta + \cos 2\theta \quad 0 \le \theta \le \pi$

a)



$x = r\cos\theta$

$x = (\theta + \cos 2\theta)(\cos\theta)$

$-2 = (\theta + \cos 2\theta)(\cos\theta)$

$-2 = \theta\cos\theta + \cos\theta\cos 2\theta$

$0 = \theta\cos\theta + \cos\theta\cos 2\theta + 2$

$\theta = 2.44$

c) $y = 1$

$y = r\sin\theta$

$y = (\theta + \cos 2\theta)\sin\theta$

$y = \theta\sin\theta + \sin\theta\cos 2\theta$

$0 = \theta\sin\theta + \sin\theta\cos 2\theta - 1$

$\theta = 1.872$

2.870

b) $x = -2$

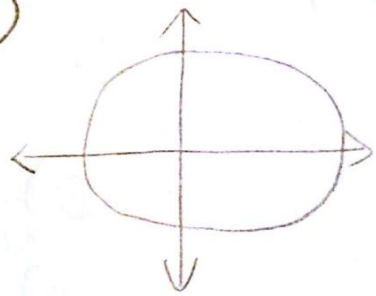
$x = r\cos\theta$

$x = [\theta + (\cos 2\theta)]\cos\theta = -2$

$0 = \theta\cos\theta + \cos\theta\cos 2\theta + 2$

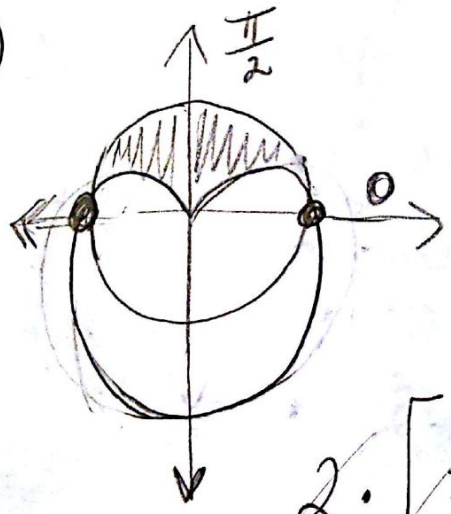
$\theta = 2.442$

13)



$$\frac{1}{2}\int_0^{2\pi} (4+2\cos\theta)^2 \, d\theta$$

15)



$2 = 2(1-\sin\theta)$

$2 = 2-2\sin\theta$

$0 = -2\sin\theta$

$\sin\theta = 0$

$\theta = 0, \pi$

$2-2\sin\theta = 0$

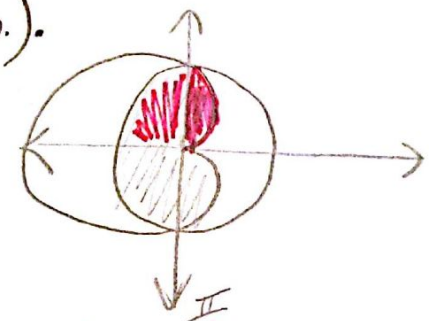
$-2\sin\theta = -2$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

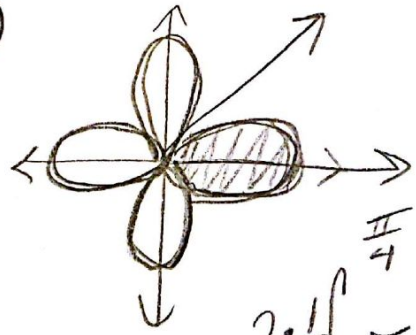
$$2\cdot\left[\frac{1}{2}\int_0^{\frac{\pi}{2}} 2^2 \, d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}} (2-2\sin\theta)^2 \, d\theta\right]$$

16).



$$2\cdot\left[\frac{1}{2}\int_0^{\frac{\pi}{2}}[2(1-\cos\theta)]^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 2^2 d\theta\right]$$

17)

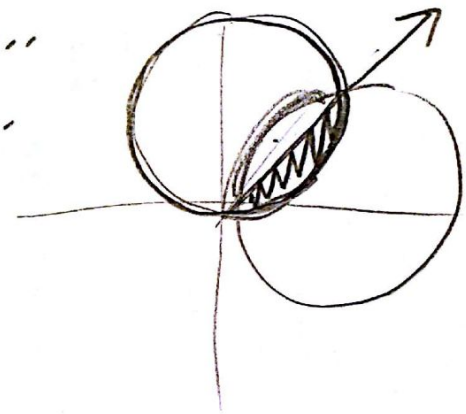


$\cos(2\theta)=0$

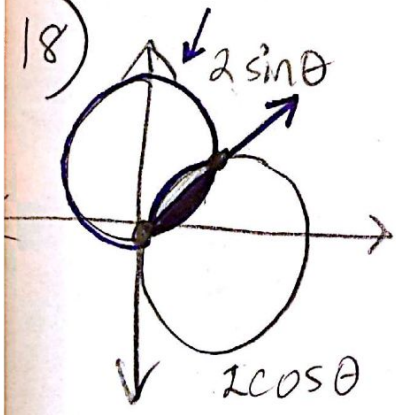
$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}\ldots$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}\ldots$

$$2\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}}[\cos(2\theta)]^2 d\theta$$



18)



$2\sin\theta = 2\cos\theta$

$\theta = \frac{\pi}{4}$

$$\frac{1}{2}\int_0^{\frac{\pi}{4}}(2\sin\theta)^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\cos\theta)^2 d\theta$$