Polar Quiz Review FRQ Rubrics:

1)

(a) Area $= \frac{1}{2}\int_0^{\pi} r^2\, d\theta$

$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2\, d\theta = 4.382$

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

(b) $-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$

$\theta = 2.786$

2 : { 1 : equation; 1 : answer }

(c) Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

2 : { 1 : information about r; 1 : information about the curve }

(d) The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

2 : { 1 : $\theta = \frac{\pi}{3}$ or 1.047; 1 : answer with justification }

2)

(a) $\text{Area} = \frac{9\pi}{4} + \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin(2\theta))^2 \, d\theta$
$= 9.708$ (or 9.707)

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) $x = (3 - 2\sin(2\theta))\cos\theta$
$\left.\frac{dx}{d\theta}\right|_{\theta=\pi/6} = -2.366$

2 : { 1 : expression for x; 1 : answer }

(c) The distance between the two curves is
$D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta)$.

$\left.\frac{dD}{d\theta}\right|_{\theta=\pi/3} = -2$

2 : { 1 : expression for distance; 1 : answer }

(d) $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3$
$\left.\frac{dr}{dt}\right|_{\theta=\pi/6} = (-2)(3) = -6$

2 : { 1 : chain rule with respect to t; 1 : answer }

3)

(a) Area $= 6\pi + \frac{1}{2}\int_{\pi/6}^{5\pi/6}(4-2\sin\theta)^2\,d\theta = 24.709$ (or 24.708)

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

(b) $x = r\cos\theta \Rightarrow x(\theta) = (4-2\sin\theta)\cos\theta$

$x(t) = \left(4-2\sin\left(t^2\right)\right)\cos\left(t^2\right)$

$x(t) = -1$ when $t = 1.428$ (or 1.427)

3 : { 1 : $x(\theta)$ or $x(t)$; 1 : $x(\theta) = -1$ or $x(t) = -1$; 1 : answer }

(c) $y = r\sin\theta \Rightarrow y(\theta) = (4-2\sin\theta)\sin\theta$

$y(t) = \left(4-2\sin\left(t^2\right)\right)\sin\left(t^2\right)$

Position vector $= \langle x(t), y(t)\rangle$

$= \left\langle \left(4-2\sin\left(t^2\right)\right)\cos\left(t^2\right), \left(4-2\sin\left(t^2\right)\right)\sin\left(t^2\right)\right\rangle$

$v(1.5) = \langle x'(1.5), y'(1.5)\rangle$

$= \langle -8.072, -1.673\rangle$ (or $\langle -8.072, -1.672\rangle$)

3 : { 2 : position vector; 1 : velocity vector }