Polar Practice Key:

① $\frac{1}{2}\int_0^{\pi/2} (\sqrt{8\sin 2\theta})^2 d\theta = \frac{1}{2}\int_0^{\pi/2} [8\sin(2\theta)] d\theta$

$u = 2\theta$
$du = 2d\theta$
$\frac{1}{2}du = d\theta$

$\frac{1}{2} \cdot 4 \cdot \int \sin u\, du = 2 \cdot \int \sin u\, du$

$= -2\cos u = -2\cos(2\theta)\Big|_0^{\pi/2}$

$= -2\cos\pi - (-2\cos 0) = \boxed{4}$

② $r = 4\sin(4\theta) = 0$

$\sin(4\theta) = 0$

$4\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$\frac{1}{2} \cdot \frac{1}{2}\int_0^{\pi/2} (4\sin(4\theta))^2 d\theta$

$\frac{1}{4}\int_0^{\pi/2} 16\sin^2(4\theta)\, d\theta$

$4\int_0^{\pi/2} \frac{1}{2}(1-\cos(8\theta))\, d\theta$

$2\int_0^{\pi/2} (1-\cos(8\theta))\, d\theta$

$2\left[\theta - \frac{1}{8}\sin(8\theta)\Big|_0^{\pi/2}\right]$

$= 2\left[\left(\frac{\pi}{2} - \frac{1}{8}\sin 4\pi\right) - 0\right]$

$= \boxed{\pi}$

(3) $7\cos\theta = 7\sin\theta$

$\cos\theta = \sin\theta$

$\cos^2\theta = 1 - \cos^2\theta$

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{\sqrt{2}}{2} \quad \theta = \frac{\pi}{4}$

$$2 \cdot \frac{1}{2}\int_0^{\pi/4} (7\sin\theta)^2 d\theta = 49\int_0^{\pi/4} (\sin^2\theta)d\theta$$

$$= \frac{1}{2} \cdot 49 \int_0^{\pi/4} (1 - \cos(2\theta))d\theta$$

$$= \frac{49}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]\Big|_0^{\pi/4} = \frac{49\pi}{8} - \frac{49}{4}$$

(4) $$\frac{1}{2}\int_{2\pi}^{3\pi} \theta^2 d\theta - \frac{1}{2}\int_0^{\pi} \theta^2 d\theta$$

$$\frac{\theta^3}{6}\Big|_{2\pi}^{3\pi} - \frac{\theta^3}{6}\Big|_0^{\pi} = 3\pi^3$$

(5) $1+\cos(3\theta)=0$

$\cos(3\theta)=-1$

$3\theta=\pi$

$\theta=\frac{\pi}{3}$

$3\cdot\frac{1}{2}\int_{-\pi/3}^{\pi/3}[1+\cos(3\theta)]^2\,d\theta$

$=4.712$

(6) $\frac{1}{2}\int_{-\pi}^{\pi}(2+2\sin\theta)^2\,d\theta = 18.850 = 6\pi$

(7) $\frac{1}{2}\int_{\pi}^{2\pi}(4+4\sin\theta)^2\,d\theta+\frac{1}{2}\int_{\pi}^{\pi}(4-4\sin\theta)^2\,d\theta$

$=11.398$

(8) $\frac{1}{2}\int_{0}^{\pi}(\sin^2\theta)^2\,d\theta = .589$