Polar HW Day 1: $\frac{dy}{dx} = \frac{r\cos\theta + \sin\theta \cdot r'}{-r\sin\theta + \cos\theta \cdot r'}$ $\begin{matrix} -c & s \\ -s & c \end{matrix}$

$x = r\cos\theta$
$y = r\sin\theta$

1) $r = 4\sin\theta, \ \theta = \frac{\pi}{3}$

$r' = 4\cos\theta$

$r\left(\frac{\pi}{3}\right) = 4\sin\frac{\pi}{3} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

$r'\left(\frac{\pi}{3}\right) = 4\cos\frac{\pi}{3} = 4 \cdot \frac{1}{2} = 2$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2\sqrt{3}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot 2}{-2\sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot 2} = \frac{\sqrt{3}+\sqrt{3}}{-3+1} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

2) $r = 3(1-\cos\theta), \ \theta = \frac{\pi}{2}$

$r = 3 - 3\cos\theta$

$r' = 3\sin\theta$

$r\left(\frac{\pi}{2}\right) = 3 - 3\cos\frac{\pi}{2} = 3 - 3(0) = 3$

$r'\left(\frac{\pi}{2}\right) = 3\sin\frac{\pi}{2} = 3(1) = 3$

$\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \frac{3(0)+1(3)}{-3(1)+0(3)} = \frac{3}{-3} = -1$$

3) $r = 2\sin(3\theta), \ \theta = \frac{\pi}{4}$

$r' = 6\cos(3\theta)$

$r\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{3\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$r'\left(\frac{\pi}{4}\right) = 6\cos\left(\frac{3\pi}{4}\right) = 6\cdot\frac{\sqrt{2}}{2} = -3\sqrt{2}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{(\sqrt{2})(\frac{\sqrt{2}}{2}) + (\frac{\sqrt{2}}{2})(-3\sqrt{2})}{(-\sqrt{2})(\frac{\sqrt{2}}{2}) + (\frac{\sqrt{2}}{2})(-3\sqrt{2})} = \frac{-1+-3}{-1+-3} = \frac{-2}{-4} = \frac{1}{2}$$

4) $r = 1 + \sin\theta$ $\qquad \frac{dy}{dx} = \frac{r\cos\theta + \sin\theta \cdot r'}{-r\sin\theta + \cos\theta \cdot r'}$

$r' = \cos\theta$

$$\frac{dy}{dx} = \frac{(1+\sin\theta)\cos\theta + \sin\theta\cos\theta}{-(1+\sin\theta)\sin\theta + \cos\theta \cdot \cos\theta}$$

$$\frac{dy}{dx} = \frac{\cos\theta + \sin\theta\cos\theta + \sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$

Horizontal:

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta(1 + 2\sin\theta) = 0$

$\cos\theta = 0 \qquad 1 + 2\sin\theta = 0$

$\theta = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}} \qquad \sin\theta = -\frac{1}{2}$

① $\qquad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

② ③

Vertical:

$-\sin\theta - \sin^2\theta + \cos^2\theta = 0$

$-\sin\theta - \sin^2\theta + (1 - \sin^2\theta) = 0$

$-\sin\theta - \sin^2\theta + 1 - \sin^2\theta = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2} \qquad \sin\theta = -1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad \theta = \cancel{\frac{3\pi}{2}}$

④ ⑤

1) $\theta = \frac{\pi}{2} \qquad r = 1 + \sin\frac{\pi}{2} = 2 \qquad x = 2\cos\frac{\pi}{2} = 0$

$y = 2\sin\frac{\pi}{2} = 2 \qquad (0, 2)$

2) $\theta = \frac{7\pi}{6} \qquad r = 1 + \sin\frac{7\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2} \qquad x = \frac{1}{2}\cos\frac{7\pi}{6} = \frac{1}{2} \cdot \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}$

$y = \frac{1}{2}\sin\frac{7\pi}{6} = \frac{1}{2} \cdot -\frac{1}{2} = -\frac{1}{4}$

$\left(-\frac{\sqrt{3}}{4}, -\frac{1}{4}\right)$

3) $\theta = \frac{11\pi}{6} \qquad r = 1 + \sin\frac{11\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2}\cos\frac{11\pi}{6} = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$y = \frac{1}{2}\sin\frac{11\pi}{6} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \qquad \left(\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$

4) $\theta = \frac{\pi}{6} \qquad r = 1 + \sin\frac{\pi}{6} = 1 + \frac{1}{2} = \frac{3}{2} \qquad x = \frac{3}{2}\cos\frac{\pi}{6} = \frac{3}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

$y = \frac{3}{2}\sin\frac{\pi}{6} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$\left(\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)$

5) $\theta = \frac{5\pi}{6} \qquad r = 1 + \sin\frac{5\pi}{6} = \frac{3}{2}$

$x = \frac{3}{2}\cos\frac{5\pi}{6} = \frac{3}{2} \cdot \frac{-\sqrt{3}}{2} = -\frac{3\sqrt{3}}{4}$

$y = \frac{3}{2}\sin\frac{5\pi}{6} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4} \qquad \left(-\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)$

5) $r = \theta + \sin 2\theta \quad 0 \le \theta \le \pi$

a) $x = -2$

$x = r\cos\theta$

$-2 = (\theta + \sin 2\theta)\cos\theta$

$\theta = 2.786$

b) $y = 1$

$y = r\sin\theta$

$1 = (\theta + \sin 2\theta)\sin\theta$

$\theta = .661$

$\theta = 2.223$

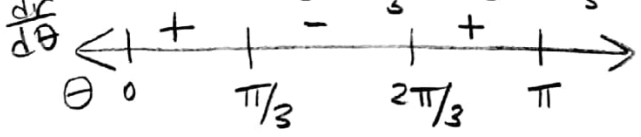
c) $\frac{dr}{d\theta} = 1 + 2\cos 2\theta = 0$

$2\cos 2\theta = -1$

$\cos(2\theta) = -\frac{1}{2}$

$2\theta = \frac{2\pi}{3} \quad 2\theta = \frac{4\pi}{3}$

$\theta = \frac{\pi}{3} \quad \theta = \frac{2\pi}{3}$



$(0, \frac{\pi}{3}) \cup (\frac{2\pi}{3}, \pi)$

6) $r=2 \quad \theta=t \quad 0\le\theta\le 2\pi$

a) $\frac{dr}{dt}=0$ particle is neither moving towards nor away from the pole

b) $\frac{dy}{dt}=0$ $\quad y=r\sin\theta$

$y=2\sin t$

$\frac{dy}{dt}=2\cos t=0$

$\cos t=0$

$t=\frac{\pi}{2},\frac{3\pi}{2}$

$(0,2),(0,-2)$

$X=r\cos\theta \quad y=r\sin\theta$

$X=2\cos\frac{\pi}{2} \quad y=2\sin\frac{\pi}{2}$

$X=0 \quad y=2$

$X=2\cos\frac{3\pi}{2} \quad y=2\sin\frac{3\pi}{2}$

$X=0 \quad y=-2$

c) $x=r\cos\theta \quad y=r\sin\theta \quad y=r\sin\theta$

$1=2\cos\theta \quad y=2\sin\frac{\pi}{3} \quad y=2\sin\frac{5\pi}{3}$

$\cos\theta=\frac{1}{2} \quad y=2\cdot\frac{\sqrt{3}}{2} \quad y=2\cdot-\frac{\sqrt{3}}{2}$

$\theta=\frac{\pi}{3},\frac{5\pi}{3} \quad y=\sqrt{3} \quad y=-\sqrt{3}$

$t=\frac{\pi}{3},\frac{5\pi}{3}$

7) $r=3+2\cos\theta$, $\theta=2t$, $0\le\theta\le 2\pi$

a) $r=3+2\cos(2t)$

$\frac{dr}{dt}=-4\sin(2t) \quad \frac{dr}{dt}\Big|_{t=3}=-4\sin 6=1.118$ away from origin

b) $r'=-2\sin\theta$

$\frac{dy}{dx}=\frac{r\cos\theta+\sin\theta\cdot r'}{-r\sin\theta+\cos\theta\cdot r'}$

$-(3+2\cos\theta)\sin\theta+\cos\theta\cdot(-2\sin\theta)=0$

$-3\sin\theta-2\sin\theta\cos\theta-2\sin\theta\cos\theta=0$

$-3\sin\theta-4\sin\theta\cos\theta=0$

$\sin\theta(-3-4\cos\theta)=0$

$(3+2\cos\theta)(\cos\theta)+\sin\theta(-2\sin\theta)$

$\sin\theta=0 \quad -4\cos\theta=3$

$\theta=0,\pi,2\pi \quad \cos\theta=-\frac{3}{4}$

$\theta=2.419,3.864$

7) b) $t = \frac{\theta}{2}$

$t = 0, \frac{\pi}{2}, \pi, 1.209, 1.932$

c) $x = 2$

$x = r\cos\theta$

$2 = (3 + 2\cos\theta)\cos\theta$

$2 = 3\cos\theta + 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = -2$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ X

$y = r\sin\theta$

$y = (3 + 2\cos\theta)\sin\theta$

$y = (3 + 2\cos\frac{\pi}{3})\sin\frac{\pi}{3}$ $\quad y = (3 + 2\cos\frac{5\pi}{3})\sin\frac{5\pi}{3}$

$y = (3 + 2\cdot\frac{1}{2})\cdot\frac{\sqrt{3}}{2}$ $\quad y = (3 + 2\cdot\frac{1}{2})\cdot -\frac{\sqrt{3}}{2}$

$y = 2\sqrt{3}$ $\quad y = -2\sqrt{3}$

d) $y = r\sin\theta$

$y = (3 + 2\cos\theta)\sin\theta$

$y = (3 + 2\cos 2t)\sin 2t$

$y = 3\sin 2t + 2\sin 2t\cos 2t$

$\left.\frac{dy}{dt}\right|_{t=\pi} = 10$ moving away from y-axis

e) $x = r\cos\theta$

$x = (3 + 2\cos\theta)\cos\theta$

$x = (3 + 2\cos 2t)\cos 2t$

$x = 3\cos 2t + 2\cos^2 2t$

$\left.\frac{dx}{dt}\right|_{t=\pi} = 0$ on the x-axis