POLAR:

The polar curve r is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

(c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

PARAMETRIC:

A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \ge 0$ with initial condition $x(0) = -4$.

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(a) Area $= \frac{1}{2}\int_{\pi/2}^{\pi} (r(\theta))^2 \, d\theta = 47.513$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(b) $-3 = r(\theta)\cos\theta = (3\theta + \sin\theta)\cos\theta$

$\theta = 2.01692$

$y = r(\theta)\sin(\theta) = 6.272$

$3 : \begin{cases} 1 : \text{equation} \\ 1 : \text{value of } \theta \\ 1 : y\text{-coordinate} \end{cases}$

(c) $y = r(\theta)\sin\theta = (3\theta + \sin\theta)\sin\theta$

$$\left.\frac{dy}{dt}\right|_{\theta=2\pi/3} = \left[\frac{dy}{d\theta}\cdot\frac{d\theta}{dt}\right]_{\theta=2\pi/3} = -2.819$$

The y-coordinate of the particle is decreasing at a rate of 2.819.

$3 : \begin{cases} 1 : \text{uses chain rule} \\ 1 : \text{answer} \\ 1 : \text{interpretation} \end{cases}$

1998 CALCULUS BC

2\.

a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\,dt = \sqrt{2t+1} + C$ (using u-substitution.)

(2 points: set up, antiderivative with +C)

Since $x(0) = -4 \Rightarrow \sqrt{1} + C = -4 \Rightarrow C = -5$.

So $x(t) = \sqrt{2t+1} - 5$ *(1 point)*

b) Method #1: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (3x^2 - 3) \cdot \frac{1}{\sqrt{2t+1}}$

Since $x(t) = \sqrt{2t+1} - 5 \Rightarrow \frac{dy}{dt} = \frac{3 \cdot \left(\sqrt{2t+1} - 5\right)^2 - 3}{\sqrt{2t+1}}$ *(2 points)*

Method #2: Implicit differentiation: $\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} - 3 \cdot \frac{dx}{dt}$

Since $\left.\begin{array}{l} x(t) = \sqrt{2t+1} - 5 \\ \frac{dx}{dt} = \frac{1}{\sqrt{2t+1}} \end{array}\right\} \Rightarrow \frac{dy}{dt} = \frac{3 \cdot \left(\sqrt{2t+1} - 5\right)^2 - 3}{\sqrt{2t+1}}$

Method #3: Since $x(t) = \sqrt{2t+1} - 5 \Rightarrow y = \left(\sqrt{2t+1} - 5\right)^3 - 3\left(\sqrt{2t+1} - 5\right)$

Taking derivatives: $\frac{dy}{dt} = 3 \cdot \left(\sqrt{2t+1} - 5\right)^2 \cdot \frac{1}{\sqrt{2t+1}} - 3 \cdot \frac{1}{\sqrt{2t+1}} = \frac{3 \cdot \left(\sqrt{2t+1} - 5\right)^2 - 3}{\sqrt{2t+1}}$

$x(4) = \sqrt{2(4)+1} - 5 = -2$

So: $y = (-2)^3 - 3(-2) = -2 \Rightarrow$ At $t = 4$: $(-2, -2)$ *(1 point)*

At $t = 4$: $\frac{dx}{dt} = \frac{1}{3}$ and $\frac{dy}{dt} = \frac{3 \cdot (3-5)^2 - 3}{3} = 3$ *(1 point)*

So: $speed = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \frac{\sqrt{82}}{3} \approx 3.018$

(2 points: speed formula, answer)

(a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\,dt$

$x(t) = \sqrt{2t+1} + C$

$x(0) = -4 = 1 + C \implies C = -5$

$x(t) = \sqrt{2t+1} - 5$

3 $\begin{cases} 1:\ x(t) = \int \frac{dt}{\sqrt{2t+1}} \\ 1:\ x(t) = \sqrt{2t+1} + C \\ 1:\ \text{evaluates } C \end{cases}$

(b) $y = x^3 - 3x$

$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 3\frac{dx}{dt}$

$= (3x^2 - 3)\frac{dx}{dt}$

$= \left[3\left(\sqrt{2t+1} - 5\right)^2 - 3\right]\left[\frac{1}{\sqrt{2t+1}}\right]$

2: answer

<−1> each error

Note: failure to express $\frac{dy}{dt}$ solely in terms of t is a single error

(c) $x(4) = \sqrt{9} - 5 = -2$

$y(4) = (-2)^3 - 3(-2) = -2$

Location at $t = 4$ is $(-2, -2)$

$\left.\frac{dx}{dt}\right|_{t=4} = \frac{1}{3}$

$\left.\frac{dy}{dt}\right|_{t=4} = \frac{3(3-5)^2 - 3}{3} = 3$

$\text{Speed} = \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{82}{9}} = 3.018$

4 $\begin{cases} 1:\ \text{position} \\ 1:\ \text{evaluates } \frac{dx}{dt} \text{ and } \frac{dy}{dt} \text{ at } t = 4 \\ 1:\ \text{uses speed formula} \\ 1:\ \text{answer} \end{cases}$

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