Related Rates Work (MC)

1) $w = x^2 y$ $\frac{dx}{dt} = -1$ $\frac{dy}{dt} = 4$ $\frac{dw}{dt} = ?$ $x = 6$ $y = 20$

product rule!

$$\frac{dw}{dt} = x^2 \frac{dy}{dt} + y\,2x \frac{dx}{dt}$$

$$\frac{dw}{dt} = (6)^2 \cdot 4 + 20 \cdot 2 \cdot 6 \cdot -1$$

$$\frac{dw}{dt} = -96 \text{ units/min}$$ **C**

2) $A = \pi r^2$ $C = 2\pi r$ $\frac{dA}{dt} = 2\frac{dC}{dt}$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$2\pi r \frac{dr}{dt} = 2\left(2\pi \frac{dr}{dt}\right)$

$2\pi r \frac{dr}{dt} = 4\pi \frac{dr}{dt}$

$2\pi r = 4\pi$ **D**

$r = 2$

3) $\tan\theta = \frac{h}{8}$ $b = 8$ $\frac{d\theta}{dt} = .03$ $h = 13$ $\theta = ?$ $\frac{dh}{dt} = ?$

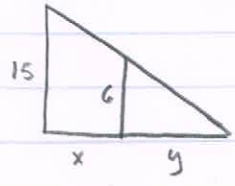
$\tan^{-1}\left(\frac{13}{8}\right) = 1.019 = \theta$

$\tan\theta = \frac{h}{8}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{8}\frac{dh}{dt}$

$\frac{1}{\cos(1.019)} \cdot \frac{1}{\cos(1.019)} \cdot .03 \cdot 8 = \frac{dh}{dt} = .874$ **E**

4)



$\frac{dx}{dt} = 4$ ft/s

$\frac{15}{x+y} = \frac{6}{y}$ $15y = 6x + 6y$

$15\frac{dy}{dt} = 6\frac{dx}{dt} + 6\frac{dy}{dt}$ $9\frac{dy}{dt} = 6(4)$

$\frac{9\frac{dy}{dt}}{9} = \frac{24}{9}$

$\frac{dy}{dt} = 2.667$ **B**

5) $V = \frac{1}{3}\pi r^2 (10 - r)$ $\frac{dr}{dt} = 6$

$V = \frac{10}{3}\pi r^2 - \frac{1}{3}\pi r^3$

$\frac{dV}{dt} = \frac{20}{3}\pi r\left(\frac{dr}{dt}\right) - \pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 40\pi r - 6\pi r^2$ **A**

Related Rates Work (FRQ)

a) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$ ← $\begin{cases} d = 8/2 \rightarrow r = 4 \\ \frac{4}{12} = \frac{r}{h} \quad r = \frac{h}{3} \end{cases}$

$V = \frac{1}{3}\pi \left(\frac{h^2}{9}\right) h$

$V = \frac{\pi}{27} h^3$

1 : $r = \frac{h}{3}$

1 : expression for volume

b) $\frac{dV}{dt} = \frac{\pi}{27} \cdot 3 \cdot h^2 \cdot \frac{dh}{dt}$ $\quad \frac{dh}{dt} = (h - 12)$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3 \cdot 3^2 \cdot (3 - 12)$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3 \cdot 9 \cdot -9$

$\frac{dV}{dt} = -28.274 \text{ ft}^3/\text{min}$ or $-9\pi \text{ ft}^3/\text{min}$

1 : $\frac{dh}{dt} = h - 12$

1 : derivative for the expression for volume

1 : answer w/ correct units

c) $V = \pi r^2 y$ $\quad \pi r^2 = 400\pi$ (area given)

$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$

$28.274 = 400\pi \frac{dy}{dt}$ or $9\pi = 400\pi \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9}{400} \text{ ft/min}$ or $\frac{dy}{dt} = .0225 \text{ ft/min}$

1 : $V = \pi r^2 y$

1 : $\pi r^2 = 400\pi$

1 : derivative for the expression for volume

1 : answer w/ correct units