

# odds

# Key

## Integration by partial fractions

1.  $\int \frac{x}{x-6} dx$

$u = x-6 \rightarrow x = u+6$   
 $du = dx$

$\int \frac{u+6}{u} du = \int 1 + \frac{6}{u}$   
 $u + 6 \ln|u| + c$

$x - 6 + 6 \ln|x-6| + c$

2.  $\int \frac{x-9}{(x+5)(x-2)} dx$

$\frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$

$x-9 = A(x-2) + B(x+5)$

$x=2, -7 = B(7), B = -1$

$x=-5, -14 = A(-7), A = 2$

$\int (\frac{2}{x+5} + \frac{-1}{x-2}) dx$

$2 \ln|x+5| - \ln|x-2| + c$

3.  $\frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$

$1 = A(x-1) + B(x+4)$

$x=1: 1 = B(5) B = 1/5$

$x=-4: 1 = A(-5), A = -1/5$

$-\frac{1}{5} \int \frac{1}{x+4} dx + \frac{1}{5} \int \frac{1}{x-1} dx$

$-\frac{1}{5} \ln|x+4| + \frac{1}{5} \ln|x-1| + c$

4.  $\int \frac{1}{(x-1)(x+1)} dx$

$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$1 = A(x+1) + B(x-1)$

$x=-1, 1 = A(-2) + B(0) \rightarrow B = -1/2$   
 $x=1, 1 = A(2) + B(0) \rightarrow A = 1/2$

$\frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + c$

5.  $\frac{x-1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x-1 = A(x+2) + B(x+1)$

$x=-2, -3 = B(-1) \rightarrow B = 3$

$x=-1, -2 = A(1) \rightarrow A = -2$

$3 \ln|x+2| - 2 \ln|x+1| + c$

6.

$x^2 - x - 6 \sqrt{\frac{x+1}{x^3 + 0x^2 - 4x - 10}}$   
 $-\frac{x^3 + x^2 + 6x}{x^2 + 2x - 10}$   
 $-\frac{x^2 + x + 6}{3x - 4}$

$\int x+1 dx + \int \frac{3x-4}{(x-3)(x+2)} dx$

$\frac{1}{2}x^2 + x + 3x-4 = A(x+2) + B(x-3)$

$x=-2, -10 = B(-5) \rightarrow B = 2$

$x=3, 5 = A(5) \rightarrow A = 1$

$\frac{1}{2}x^2 + x + \ln|x-3| + 2 \ln|x+2| + c$

7.  $\int \frac{3x}{x(x-5)} dx = \int \frac{3}{x-5} dx$

$3 \ln|x-5| + c$

8.  $\int \frac{x^2}{x+4} dx$   $x+4 \sqrt{\frac{x-4}{x^2+0x+0}}$   
 $-\frac{x^2+4x}{x^2+4x}$

$\int x-4 dx + \int \frac{16}{x+4} dx$   $-\frac{4x+0}{x^2+4x}$   
 $+\frac{4x+16}{16}$

$\frac{1}{2}x^2 - 4x + 16 \ln|x+4| + c$

9.  $4x^2 - 7x - 12 = A(x)(x+2) + B(x)(x-3) + C(x+2)(x-3)$

$x=0, -12 = C(-6) \rightarrow C = 2$

$x=-2, 18 = 40B \rightarrow B = 1.8$  or  $\frac{9}{5}$

$x=3, 3 = 15A \rightarrow A = 1/5$

$2 \ln|x| + 9/5 \ln|x+2| + 1/5 \ln|x-3| + c$

\* 10. use formulas  
 $\int \frac{\sqrt{x+4}}{x} dx = 2\sqrt{x+4} + 4 \int \frac{dx}{x\sqrt{x+4}}$   
 $2\sqrt{x+4} + 4 \frac{1}{2} \ln \left| \frac{\sqrt{x+4}-2}{\sqrt{x+4}+2} \right| + c$   
 $2\sqrt{x+4} + 2 \ln \left| \frac{\sqrt{x+4}-2}{\sqrt{x+4}+2} \right| + c$

20.  $\int \frac{dx}{x(ax+b)}$   
 $1 = Ax + B(ax+b)$   
 $x=0, 1 = B(b), B = \frac{1}{b}, x = -\frac{b}{a}, 1 = A(-\frac{b}{a}) A = -\frac{a}{b}$

deriv  $\frac{-a}{b}$   
 $\frac{-a}{b} \frac{1}{ax+b}$   
 $\frac{-a}{b} \frac{1}{a} \ln|ax+b|$

11.  $\int \frac{x+2}{x(x+1)} dx$

$x+2 = A(x+1) + Bx$

$x=-1, 1 = -1B, B = -1$

$x=0, 2 = 1A, A = 2$

$2 \ln|x| - \ln|x+1| + C$

12.  $\frac{1}{(x-5)(x-3)} = \frac{A}{x-5} + \frac{B}{x-3}$

$1 = A(x-3) + B(x-5)$

$x=3, 1 = -2B, B = -\frac{1}{2}$

$x=5, 1 = 2A, A = \frac{1}{2}$

$\frac{1}{2} \ln|x-5| - \frac{1}{2} \ln|x-3| + C$

13.  $\frac{x+2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$

$x+2 = A(x+1) + B(2x+1)$

$x=-1, 1 = -1B, B = -1$

$x = -\frac{1}{2}, \frac{3}{2} = \frac{1}{2}A, A = 3$

$\frac{3}{2} \ln|2x+1| - \ln|x+1| + C$

deriv  $\frac{3}{2x+1}$   
 $\frac{1}{2} (3) \ln|2x+1|$

14.  $\int \frac{x}{(x-2)(x-1)} dx$

$x = A(x-1) + B(x-2)$

$x=1, 1 = B(-1), B = -1$

$x=2, 2 = A(1), A = 2$

$2 \ln|x-2| - \ln|x-1| + C$

15.

$x^2 + 3x + 2 \int \frac{x^2}{(x+2)(x+1)} dx$

$\int x^2 dx + \int \frac{1}{(x+2)(x+1)} dx$   
 $1 = A(x+1) + B(x+2)$

$x=-1, 1 = B, B = 1$   
 $x=-2, 1 = -A, A = -1$

$\frac{1}{3}x^3 + \ln|x+1| - \ln|x+2| + C$

16.  $\int \frac{1}{(x+4)(x+1)} dx$

$1 = A(x+1) + B(x+4)$

$x=-1, 1 = A + B(3), B = \frac{1}{3}$

$x=-4, 1 = A(-3), A = -\frac{1}{3}$

$-\frac{1}{3} \ln|x+4| + \frac{1}{3} \ln|x+1| + C$

17.

$x^2 + 3x + 2 \int \frac{x-3}{(x^3+0x^2+0x+0) - (x^3+3x^2+2x)}$

$3x^2 - 2x + 0$   
 $3x^2 + 9x + 6$   
 $7x + 6$

$\int x-3 dx + \int \frac{7x+6}{(x+1)(x+2)} dx$

$7x+6 = A(x+2) + B(x+1)$

$x=-2, -8 = B(-1), B = 8$

$x=-1, -1 = A$

$\frac{1}{2}x^2 - 3x - \ln|x+1| + 8 \ln|x+2| + C$

18.

$x^2 - 3x + 2 \int \frac{1}{(x^2+0x+1) - (x^2+3x+2)}$

$\int dx + \int \frac{3x-1}{(x-2)(x-1)} dx$

$3x-1 = A(x-1) + B(x-2)$

$x=1, 2 = B(-1), B = -2$

$x=2, 5 = A(1), A = 5$

$x + 5 \ln|x-2| - 2 \ln|x-1| + C$

19.

$\int \frac{dx}{x(x+1)}$

$1 = A(x+1) + B(x)$

$x=0, 1 = A$

$x=-1, 1 = -B, B = -1$

$\ln|x| - \ln|x+1| + C$

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