PARTNER A:

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \sin(t^3)$, $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at the position $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 2$.

(b) Write the equation of the tangent line to the curve at the point where $t = 2$.

(c) Find the speed of the vector at $t = 2$.

(d) Find the position of the particle at time $t = 1$.

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(c) Find the speed of the vector at $t = 2$.

(d) Find the position of the particle at time $t = 1$.

PARTNER B:

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \sin(t^3 - t)$ and $\frac{dy}{dt} = \cos(t^3 - t)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

(c) Find the magnitude of the velocity vector at $t = 3$.

(d) Find the position of the particle at time $t = 2$.

PARTNER B:

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \sin(t^3 - t)$ and $\frac{dy}{dt} = \cos(t^3 - t)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

(c) Find the magnitude of the velocity vector at $t = 3$.

(d) Find the position of the particle at time $t = 2$.

A KEY:

Solution:

(a) Students should use their calculators to numerically differentiate both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ when $t = 2$ to get $a(2) =$ <-1.746, 3.027>

(b) When $t = 2$, $\frac{dy}{dx} = \frac{\cos 4}{\sin 8}$ or -0.661, so the tangent line equation is

$y - 4 = \frac{\cos 4}{\sin 8}(x-1)$ or $y - 4 = -0.661(x-1)$.

Notice that it is fine to leave the slope as the exact value or to write it as a decimal correct to three decimal places.

(c) Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(\sin 8)^2 + (\cos 4)^2}$ or 1.186

Notice that it is fine to leave the speed as the exact value or to write it as a decimal correct to three decimal places.

(d) Students should apply the Fundamental Theorem of Calculus to find the *x* and *y* components of the position.

$$x(1) = x(2) - \int_1^2 x'(t)\,dt \qquad y(1) = y(2) - \int_1^2 y'(t)\,dt$$

$$= 1 - \int_1^2 \sin(t^3)\,dt \qquad = 4 - \int_1^2 \cos(t^2)\,dt$$

$$= 0.782 \qquad = 4.443$$

Therefore the position at time $t = 1$ is (0.782, 4.443).

B KEY:

(a) $a(3) = \langle 11.029, 23.545 \rangle$

(b)

$$\frac{dy}{dx} = \left.\frac{\cos(t^3 - t)}{\sin(t^3 - t)}\right|_{t=3} = -0.468 \text{ so the tangent line equation is } y - 4 = -0.468(x - 1)$$

(c) Magnitude $= \left.\sqrt{\left(\sin(t^3 - t)\right)^2 + \left(\cos(t^3 - t)\right)^2}\right|_{t=3} = 1$

(d) $x(2) = 1 - \int_2^3 \sin(t^3 - t)\,dt = 0.932$, $y(2) = 4 - \int_2^3 \cos(t^3 - t)\,dt = 4.002$ so the position = (0.932, 4.002)