

Olympics Review:

- 1) $x = \sqrt{t}$ $t = x^2$
 $y = 3t^2 + 2t$ $y = 3(x^2)^2 + 2(x^2)$
 $y = 3x^4 + 2x^2$
- 2) $x = \ln t$ $t = e^x$
 $y = t^2 + t$ $y = (e^x)^2 + e^x$
 $y = e^{2x} + e^x$
- 3) $x = 3\sin t + 2$ $x - 2 = 3\sin t$ $y + 1 = 4\cos t$
 $y = 4\cos t - 1$ $\frac{x-2}{3} = \sin t$ $\frac{y+1}{4} = \cos t$
 $\frac{(x-2)^2}{9} = \sin^2 t$ $\frac{(y+1)^2}{16} = \cos^2 t$
 $\sin^2 t + \cos^2 t = \frac{(x-2)^2}{9} + \frac{(y+1)^2}{16}$
 $1 = \frac{(x-2)^2}{9} + \frac{(y+1)^2}{16}$
- 4) $x = \sqrt{t} = t^{1/2}$ $\frac{dy}{dx} = \frac{6t+2}{\frac{1}{2}t^{-1/2}} = (6t+2)(2t^{1/2})$
 $y = 3t^2 + 2t$ $= 12t^{3/2} + 4t^{1/2}$
- 5) $x = \ln t$ $\frac{dy}{dx} = \frac{2t+1}{\frac{1}{t}} = (2t+1)t = 2t^2 + t$
 $y = t^2 + t$
- 6) $x = 3\sin t + 2$ $\frac{dy}{dx} = \frac{-4\sin t}{3\cos t} = -\frac{4}{3} \tan t$
 $y = 4\cos t - 1$

$$7) \frac{18t^{1/2} + 2t^{-1/2}}{\frac{1}{2}t^{-1/2}} = (18t^{1/2} + 2t^{-1/2})(2t^{1/2})$$

$$= 36t + 4$$

$$8) \frac{4t+1}{\frac{1}{t}} = 4t^2 + t$$

$$9) \frac{-\frac{4}{3} \sec^2 t}{3 \cos t} = -\frac{4}{9} \sec^3 t$$

$$10) \frac{dy}{dx} = \frac{2t-4}{1} \quad \text{HT: } 2t-4=0 \quad x=7$$

$$t=2 \quad y=-4$$

$$\text{VT: none}$$

$$11) \frac{dy}{dx} = \frac{3t^2-3}{2t-1} \quad \text{HT: } 3t^2-3=0 \quad t=1: x=1$$

$$t=\pm 1 \quad y=-2$$

$$t=-1: x=3$$

$$y=2$$

$$\text{VT: } 2t-1=0 \quad x=\frac{3}{4}$$

$$t=\frac{1}{2} \quad y=-\frac{11}{8}$$

$$12) \frac{dy}{dx} = \frac{4\cos t}{-2\sin t} = \frac{-2\cos t}{\sin t}$$

$$\text{HT: } -2\cos t = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$t = \frac{\pi}{2}; x=3$$

$$y=3$$

$$\text{VT: } \sin t = 0$$

$$t = 0, \pi$$

$$t=0; x=5$$

$$y=-1$$

$$t=\pi; x=1$$

$$y=-1$$

$$t = \frac{3\pi}{2}; x=3$$

$$y=-5$$

$$13) \int_0^2 \sqrt{(2t)^2 + (3t^2)^2} dt \approx 9.073$$

$$14) \int_{-2}^2 \sqrt{(2e^{2t})^2 + 3^2} dt \approx 60.725$$

$$15) \int_0^{2\pi} \sqrt{(6(\cos t)^2 - \sin t)^2 + (6(\sin t)^2 \cdot \cos t)^2} dt = 12$$

$$16) v(t) = \langle t \cdot \cos t + \sin t, -t \sin t + \cos t \rangle$$

$$17) v(t) = \langle -te^{-t} + e^{-t}, -e^{-t} \rangle$$

$$18) v(t) = \langle 2t + 2\cos 2t, 2t + 2\sin 2t \rangle$$

$$19) x(3) = x(0) + \int_0^3 (t+1)^{-1} dt \approx 4.386$$

$$y(3) = y(0) + \int_0^3 (t+2)^{-2} dt = -1.7$$

$$20) x(3) = 2 + \int_0^3 (3t^2 - 2t) dt = 20$$

$$y(3) = 6 + \int_0^3 (1 + \cos \pi t) dt = 9$$

$$21) x(3) = 1 + \int_0^3 (e^t - t) dt \approx 15.586$$

$$y(3) = 1 + \int_0^3 (e^t + t) dt \approx 24.586$$

$$22) \quad v(t) = \langle \pi \sec \pi t \tan \pi t, \pi \sec^2(\pi t) \rangle$$

$$v\left(\frac{1}{4}\right) = \langle \pi \sec \frac{\pi}{4} \tan \frac{\pi}{4}, \pi \sec^2\left(\frac{\pi}{4}\right) \rangle$$

$$v\left(\frac{1}{4}\right) = \langle \sqrt{2}\pi, 2\pi \rangle$$

$$\text{speed} = \sqrt{(\sqrt{2}\pi)^2 + (2\pi)^2}$$

$$\sqrt{2\pi^2 + 4\pi^2}$$

$$\sqrt{6\pi^2} = \sqrt{6}\pi \approx 7.695$$

$$23) \quad v = 0, \quad 0 \leq t \leq 2\pi$$

$$v(t) = \langle \cos t, -2 \sin(2t) \rangle$$

$$\cos t = 0$$

$$t = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

$$-2 \sin(2t) = 0$$

$$\sin(2t) = 0$$

$$2t = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$t = 0, \left(\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right)$$

$$24) \quad \int_0^1 \sqrt{(e^t \cos t + \sin t e^t)^2 + (e^t \sin t + \cos t e^t)^2} dt$$

$$\approx 2.430$$