1 1992 BC #3 : $x = e^t \sin t \quad y = e^t \cos t$

a) $\frac{dy}{dt} = e^t(-\sin t) + e^t \cos t = e^t(\cos t - \sin t)$

$\frac{dx}{dt} = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$$

$$t = \frac{\pi}{2} \qquad \frac{dy}{dx} = \frac{\cos\frac{\pi}{2} - \sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + \sin\frac{\pi}{2}} = \boxed{-1}$$

b) $$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{e^{2t}(\cos t + \sin t)^2 + e^{2t}(\cos t - \sin t)^2}$$

$$= e^t \sqrt{\cancel{\cos^2 t} + \cancel{2\cos t \sin t} + \cancel{\sin^2 t} + \cos^2 t \cancel{- 2\cos t \sin t} \cancel{+ \sin^2 t}}$$

$$= e^t \sqrt{1+1} = e^t \sqrt{2}$$

$t = 1$, speed is $\boxed{e\sqrt{2}}$

c) $$\text{dist.} = \int_0^1 e^t \sqrt{2}\, dt = e^t \sqrt{2} \Big|_0^1$$

$$e\sqrt{2} - \sqrt{2} = (e-1)\sqrt{2}$$

$$= \boxed{\sqrt{2}(e-1)}$$

2003 #2:

a) At point C, dy/dt is not positive because $y(t)$ is decreasing along arc BD as t increases.
At point C, dx/dt is not positive because $x(t)$ is decreasing along arc BD as t increases.

b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \text{undef} \therefore \frac{dx}{dt} = 0$

$-9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right) = 0$

$-9\cos\frac{\pi t}{6} = 0$ $\quad$ $\sin\frac{\pi\sqrt{t+1}}{2} = 0$

$\frac{\pi t}{6} = \frac{\pi}{2}$ $\quad$ $\frac{\pi\sqrt{t+1}}{2} = \pi$

$\pi\sqrt{t+1} = 2\pi$

$\sqrt{t+1} = 2$

$t+1 = 4$

$t = 3$

c) $x'(8) = -9\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{3\pi}{2}\right) = -\frac{9}{2}$

$\frac{dy}{dx} = \frac{y'(8)}{x'(8)} = \frac{5}{9}$

$y'(8) = \frac{5}{9}x'(8) = -\frac{5}{2}$

$\vec{v} = \langle -4.5, -2.5 \rangle$

Speed $= \sqrt{(4.5)^2 + (2.5)^2} = 5.147$

d) $x(9) - x(0) = \int_0^9 x'(t)\,dt = -39.255$

39.255 ft apart

2004 #3:

$$\frac{dx}{dt} = 3 + \cos(t^2)$$

$$t = 2,\ (1,8)$$

a) $$x(4) = \underset{1}{x(2)} \underset{+}{+} \int_2^4 \left(3 + \cos(t^2)\right) dt$$

$$= 7.132$$

b) $$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-7}{3 + \cos 4} = -2.983$$

$$y - 8 = -2.983(x - 1)$$

c) $$\sqrt{(x'(2))^2 + (y'(2))^2} = 7.832$$

d) $$x''(t) = -\sin(t^2) \cdot 2t$$

$$x''(4) = -\sin 16 \cdot 8$$

$$x''(4) = 2.303$$

$$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t+1)(3 + \cos(t^2))$$

$$y''(t) = (2t+1)(-\sin(t^2) \cdot 2t) + (3 + \cos(t^2))(2)$$

$$y''(4) = 24.813$$

$$a(t) = < 2.303, 24.813 >$$

2010 BC #2B:

$$\frac{dx}{dt} = 14\cos(t^2)\sin(e^t)$$

$$0 \le t \le 1.5$$

$$\frac{dy}{dt} = 1 + 2\sin(t^2)$$

$t=0$, $(-2, 3)$

a) $x'(t) = 0$

$y'(t) \neq 0$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$x'(t) = 14\cos(t^2)\sin(e^t) = 0$

$14\cos(t^2) = 0$

$\cos t^2 = 0$

$t^2 = \frac{\pi}{2}$

$\boxed{t \approx 1.253}$

$\sin(e^t) = 0$

$e^t = 0 \quad t = \emptyset$

$e^t = \pi \quad t = \ln \pi$

$\boxed{t \approx 1.145}$

b) $\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)} = 0.863$

$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.315$

$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.621$

$\boxed{y = 4.621 + 0.863(x - 9.315)}$

c)

$\sqrt{(x'(1))^2 + (y'(1))^2} = \boxed{4.105}$

d) $< x''(1), y''(1) >$

$\boxed{< -28.425, 2.161 >}$

BC 2000 #4:

$(x(t), y(t))$ $t=1$, $(2,6)$

$\vec{v} = \langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \rangle$

$= \langle 1 - t^{-2}, 2 + t^{-2} \rangle$

a) $a(t) = \langle \frac{2}{t^3}, \frac{-2}{t^3} \rangle$ (1)

$a(3) = \langle \frac{2}{27}, \frac{-2}{27} \rangle$ (1)

(1) antider.

b) $\langle t + \frac{1}{t} + c, 2t - \frac{1}{t} + c \rangle$

$2 = 1 + 1 + c$ (1) uses initial cond. $6 = 2 - 1 + c$

$c = 0$ $c = 5$

$\langle t + \frac{1}{t}, 2t - \frac{1}{t} + 5 \rangle$

$t = 3$: $\left(\frac{10}{3}, \frac{32}{3}\right)$ (1)

c) $\frac{dy}{dx} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$ (1) eqn

$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right)$

$2 + \frac{1}{t^2} = 8 - \frac{8}{t^2}$

$\frac{9}{t^2} = 6$

$\frac{6t^2}{6} = \frac{9}{6}$

$t^2 = \frac{3}{2}$

$t = \sqrt{\frac{3}{2}}$ (1)

d) $\lim_{t \to \infty} \frac{dy}{dx} =$ (1) limit

$\lim_{t \to \infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

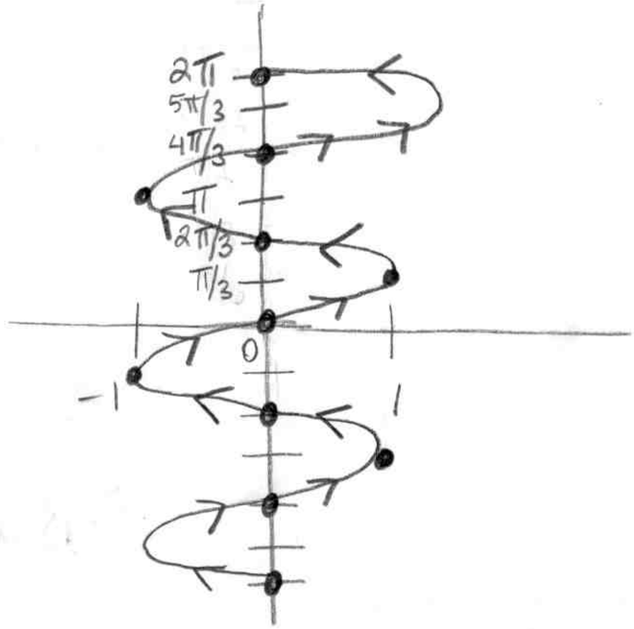
L'H: $\frac{2t^2 + 1}{t^2 - 1}$

$\frac{4t}{2t} = 2$ (1)

BC 2002B #1:

a) $x(t) = \sin(3t)$ $y(t) = 2t$ $-\pi \le t \le \pi$

t	x	y
$-\pi$	0	-2π
$-\pi/2$	1	$-\pi$
$-\pi/3$	0	$-2\pi/3$
$-\pi/6$	-1	$-\pi/3$
0	0	0
$\pi/6$	1	$\pi/3$
$\pi/3$	0	$2\pi/3$
$\pi/2$	-1	π
$2\pi/3$	0	$4\pi/3$
π	0	2π



(1) graph
* 3 cycles of sine
* x between -1 and 1
* y between -2π and 2π

(1) direction

b) $-1 \le x(t) \le 1$ (1) $-2\pi \le y(t) \le 2\pi$ (1)

c) $x(t) = \sin(3t)$ $y(t) = 2t$

$x'(t) = 3\cos(3t)$ $y'(t) = 2$

$3\cos(3t) = 0$ (1)

$\cos(3t) = 0$

$3t = \cos^{-1}(0)$

$3t = \frac{\pi}{2}$

$t = \frac{\pi}{6}$ (1)

$\sqrt{(3\cos(3t))^2 + 2^2}$ with $t = \frac{\pi}{6}$

$\sqrt{4} = 2$ (1)

d) $\int_{-\pi}^{\pi} \sqrt{(3\cos(3t))^2 + 2^2}\, dt$ (1)

≈ 17.973 (1) $> 5\pi$