AP Calculus – Optimization Problems 13 Name: KEY

1. A point moves on the x-axis in such a way that its velocity at time t, ($t > 0$) is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum? (No calculator)

$v = \frac{1}{t} \cdot \ln t$

A. 1 B. $e^{\frac{1}{2}}$ C. e D. $e^{\frac{3}{2}}$ E. There is no maximum value for v.

$a = \frac{1}{t} \cdot \frac{1}{t} + \ln t\left(-\frac{1}{t^2}\right) = \frac{1}{t^2} + \ln t\left(-\frac{1}{t^2}\right) =$

$\frac{1}{t^2}(1 - \ln t) = 0$

$1 - \ln t = 0$

$\ln t = 1$

$t = e$

2. The derivative of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$ (No calculator)

$f'(x) = \frac{4}{3}x^3 - x^4$ $f''(x) = 4x^2 - 4x^3$

$4x^2(1 - x) = 0$

A. -1 B. 0 C. 1 D. 4/3 E. 5/3

$x = 0, 1$

$f''(x)$: $+$ at 0, $+$ at 1, $-$

3. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$ (No calculator)

$f'(x) = 3x^2 - 6x = 0$

$3x(x - 2) = 0$

$f'''(x) = 8x - 12x^2$

$f'''(1) = 8 - 12 = -4$

CC↓

A. 4 B. 2 C. 1 D. 0 E. -2

$x = 0, 2$

x	f(x)
-2	-8
0	12
2	8
4	28

$-8 - 12 + 12$

$8 - 12 + 12$

$64 - 48 + 12$

4. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (No Calculator)

A. $2\sqrt[3]{2}$ B. $2\sqrt{2}$ C. $2\sqrt[3]{4}$ D. 4 E. 8

5. If $y = 2x - 8$, what is the minimum value of the product xy? ~~(Calculator permitted)~~

A. -16 B. -8 C. -4 D. 0 E. 2

$M = (2x - 8)x$

$M = 2x^2 - 8x$

$\frac{dM}{dx} = 4x - 8 = 0$

$x = 2$ $y = -4$

6. The maximum acceleration attained on the interval, $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$. (No calculator)

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$ $t = 1$

0	4
1	14
3	40

$1 - 3 + 12 + 4$

$27 - 27 + 36 + 4$

A. 9 B. 12 C. 14 D. 21 E. 40

$V = 16\pi = \pi r^2 h$

$r^2 h = 16$

$h = \frac{16}{r^2}$

$A = 2\pi r h + 2\pi r^2$

$A = 2\pi r\left(\frac{16}{r^2}\right) + 2\pi r^2$

$A = 32\pi r^{-1} + 2\pi r^2$

$\frac{dA}{dr} = -\frac{32\pi}{r^2} + 4\pi r \cdot \frac{r^2}{r^2}$

$\frac{-32\pi + 4\pi r^3}{r^2} = 0$

$-4\pi(8 - r^3) = 0$

$8 - r^3 = 0$

$r^3 = 8$

$r = 2$

$h = \frac{16}{2^2} = 4$

$3(9) - 6(3) + 12$

$27 - 18 + 12$

$9 + 12$

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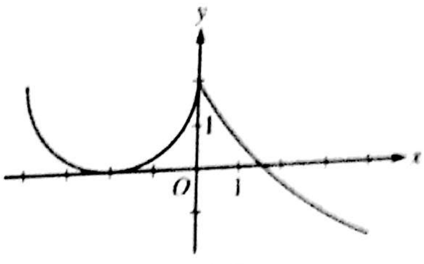
...e-Response Questions:

1. The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}.$$

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.



Graph of f'

a. For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

POI: where $f''(x)$ changes signs
$f'(x)$ goes inc to dec or dec to inc

(1) $x = -2$: $f''(x)$ goes neg to pos
$x = 0$: $f''(x)$ goes pos to neg (1) justify

b. For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

$f'(x) = 0$ at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$

On $(-4, -2)$: $f'(x)$ is pos, so f is increasing
$(-2, 0)$
$(0, 3\ln(\frac{5}{3}))$
On $(3\ln(\frac{5}{3}), 4)$: $f'(x)$ is neg, so f is decreasing

(1) Abs max at $x = 3\ln(\frac{5}{3})$ b/c f' goes from pos to neg (1)

2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

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Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(1) $R'(t) = 0$ when $t = 0$ and $t = 1.363$ (A)
(1)

$R'(t) = 2760t - 2025t^2$

t	$f(t)$
0	0
A	854.527
2	120

(1)
max rate at $t = 1.363$