OPTIMIZATION WORKSHEET

1) x - FIRST #

y - SECOND #

$xy = 192$

$y = \frac{192}{x}$

$x + 3y = S$

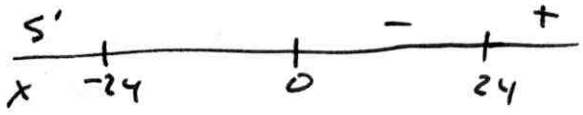
$S = x + 3\left(\frac{192}{x}\right)$

$S = x + \frac{576}{x}$

$S' = 1 - 576x^{-2} = \frac{x^2 - 576}{x^2}$

$x^2 - 576 = 0$ $\quad x^2 = 0$

$x = \pm 24$ $\quad x > 0$



HAS TO BE POSITIVE

$x = 24$

$y = \frac{192}{24} = 8$

THE NUMBERS ARE 8 AND 24.

2) x - FIRST #

y - SECOND #

$x + 2y = 100$

$y = \frac{100 - x}{2}$

$P = xy$ (MAXIMIZE)

$P = x\left(\frac{100 - x}{2}\right)$

$P = \frac{100x - x^2}{2} = 50x - \frac{x^2}{2}$

$P' = 50 - x$

$50 - x = 0$

$\boxed{x = 50}$

$50 + 2y = 100$

$2y = 50$

$\boxed{y = 25}$

THE NUMBERS ARE 25 AND 50.

3)

$x + 2y = 120$

$y = 60 - \frac{x}{2}$

$A = xy$

$A = 60x - \frac{x^2}{2}$

$A' = 60 - x$

$60 - x = 0$

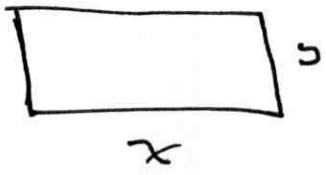
$x = 60$

$60 + 2y = 120$

$y = 30$

THE DIMENSIONS FOR GREATEST AREA ARE 30 ft BY 60 ft

4)



$2x + 2y = 80$

$y = 40 - x$

$A = x(40 - x)$

$A = 40x - x^2$

$A' = 40 - 2x$

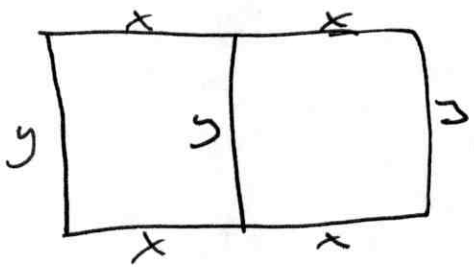
$40 - 2x = 0$

$\boxed{x = 20}$

MAX $A = 20(40 - 20)$

MAX AREA IS $400\ cm^2$

5)



$A = 2xy$

$4x + 3y = 102$

$y = \frac{102 - 4x}{3}$

$A = 2x\left(\frac{102 - 4x}{3}\right)$

$A = 68x - \frac{8}{3}x^2$

$A' = 68 - \frac{16}{3}x$

$68 - \frac{16}{3}x = 0$

$\frac{16}{3}x = 68$

$x = \frac{51}{4}m$

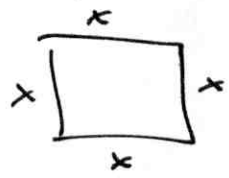
$y = \frac{102 - \frac{51}{4}(4)}{3}$

$y = 17$

$A = 2\left(\frac{51}{4}\right)(17) = 433.5\ m^2$

MAX AREA $= 433.5\ m^2$

6)

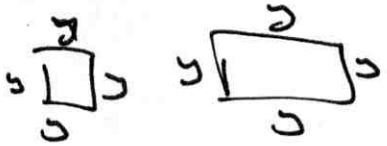


$4x = 200$

$x = 50$

$A = x^2$

$A = 2500\ m^2$



$8y = 200$

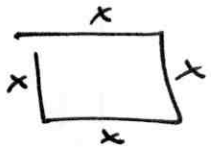
$y = 25$

$A = 2y^2$

$A = 2(25)^2$

$A = 1250\ m^2$

7)



$2\pi r + 4x = 40$

$2\pi r = 40 - 4x$

$\pi r = 20 - 2x$

$r = \frac{20-2x}{\pi}$

$A = \pi r^2 + x^2$

$A = \pi\left(\frac{20-2x}{\pi}\right)^2 + x^2$

$A = \pi\left(\frac{400 - 80x + 4x^2}{\pi^2}\right) + x^2$

$A = \frac{400 - 80x + 4x^2 + \pi x^2}{\pi}$

$A = \frac{400}{\pi} - \frac{80x}{\pi} + \frac{4x^2}{\pi} + \frac{\pi x^2}{\pi}$

$A' = \frac{-80}{\pi} + \frac{8x}{\pi} + \frac{2x\pi}{\pi} = \frac{2\pi x + 8x - 80}{\pi} = 0$

$2\pi x + 8x - 80 = 0$

$x(2\pi + 8) = 80$

$x = \frac{80}{2\pi + 8} \approx 5.601\ cm$

$r = \frac{20 - 2(5.601)}{\pi}$

$r \approx 2.800\ cm$

a) PERIMETER OF SQUARE = 22.404 cm

CIRCUMFERENCE = 17.593 cm

b) CIRCLE WITH CIRCUMFERENCE 40cm

$40 = 2\pi r$

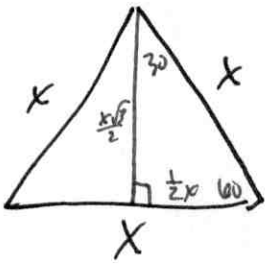
$r = \frac{20}{\pi}$

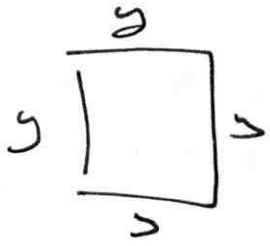
$A = \pi\left(\frac{20}{\pi}\right)^2 = \frac{400}{\pi}$

$A = 127.324\ cm^2$

8)   4 ft of wire

TO CREATE THE MAX AREA ALL FOUR FEET OF WIRE SHOULD BE USED FOR THE CIRCLE.

9) 



$3x+4y=10$

$4y=10-3x$

$y=\frac{10-3x}{4}$

$A_\triangle=\frac{1}{2}bh$

$A_\triangle=\frac{1}{2}x\left(\frac{\sqrt{3}}{2}\right)x=\frac{\sqrt{3}}{4}x^2$

$A_\square=y^2$

$A_\square=\left(\frac{10-3x}{4}\right)^2$

$A_\square=\frac{100-60x+9x^2}{16}$

$A=\frac{\sqrt{3}}{4}x^2+\frac{100}{16}-\frac{60}{16}x+\frac{9}{16}x^2$

$A'=\frac{\sqrt{3}}{2}x-\frac{15}{4}+\frac{9}{8}x$

$\frac{\sqrt{3}}{2}x-\frac{15}{4}+\frac{9}{8}x=0$

$x\left(\frac{\sqrt{3}}{2}+\frac{9}{8}\right)=\frac{15}{4}$

$x\left(\frac{4\sqrt{3}+9}{8}\right)=\frac{30}{8}$

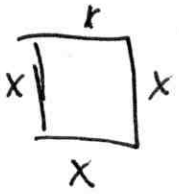
$x=\frac{30}{4\sqrt{3}+9}\approx 1.883$

$y=\frac{10-3(1.883)}{4}$

$y=1.087$

TO PRODUCE THE MAXIMUM AREA THE SIDES OF THE SQUARE SHOULD BE 1.087 AND THE SIDES OF THE TRIANGLE SHOULD BE 1.883.

10)



$A = \pi r^2 + x^2$

$2\pi r + 4x = 16$

$2\pi r = 16 - 4x$

$\pi r = 8 - 2x$

$r = \frac{8-2x}{\pi}$

$A = \pi\left(\frac{8-2x}{\pi}\right)^2 + x^2$

$A = \frac{64}{\pi} - \frac{32x}{\pi} + \frac{4}{\pi}x^2 + x^2$

$A' = -\frac{32}{\pi} + \frac{8}{\pi}x + 2x = 0$

$x\left(\frac{8+2\pi}{\pi}\right) = \frac{32}{\pi}$

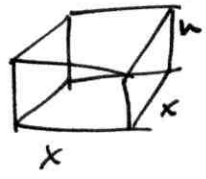
$x = \frac{32}{8+2\pi} \simeq 2.240$

$r = \frac{8-2x}{\pi}$

$r = \frac{8 - 2\left(\frac{32}{8+2\pi}\right)}{\pi}$

$r = 1.120$

11)



$SA = x^2 + 4xh = 108$

$4xh = 108 - x^2$

$h = \frac{108-x^2}{4x} = \frac{27}{x} - \frac{x}{4}$

$V = x^2 h$

$V = x^2\left(\frac{27}{x} - \frac{x}{4}\right)$

$V = 27x - \frac{x^3}{4}$

$V' = 27 - \frac{3}{4}x^2 = 0$

$27 = \frac{3x^2}{4}$

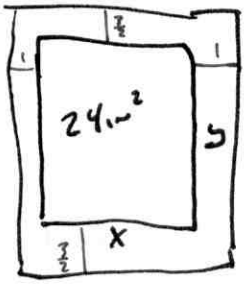
$36 = x^2$

$x = 6$ in

$h = \frac{108-36}{4(6)}$

$h = 3$ in

12)



$xy = 24$

$y = \frac{24}{x}$

$A = (x+3)(y+2)$

$A = xy + 3x + 2y + 6$

$A = 24 + 3x + \frac{48}{x} + 6$

$A' = 3 - \frac{48}{x^2} = 0$

$3 = \frac{48}{x^2}$

$x^2 = \frac{48}{3}$

$x^2 = 16$

$x = 4in$

$xy = 24$

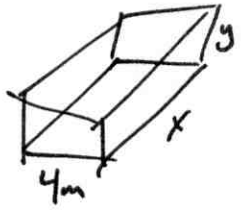
$y = \frac{24}{x}$

$y = \frac{24}{4}$

$y = 6in$

SIZE OF PAPER SHOULD BE $x+2$ OR 6 IN BY $y+3$ OR 9 in

13)



$V = 36\,m^3$

$V = 4xy = 36$

$y = \frac{9}{x}$

$COST = 4x(10) + 2(4y)(5) + 2xy(5)$

$C = 40x + 40y + 10xy$

$C = 40x + 40(\frac{9}{x}) + 10x(\frac{9}{x})$

$C = 40x + \frac{360}{x} + 90$

$C' = 40 + \frac{360}{x^2} = 0$

$40 = \frac{360}{x^2}$

$x^2 = \frac{360}{40}$

$x = 3m$

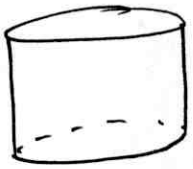
$y = \frac{9}{3}$

$y = 3\,m$

~~$COST = 40(3) + 360 + 90$~~

$COST = (3+3)(5)(2) + (3\cdot4)(5)(2) + (3\cdot4)(10)$

$COST = \$330$

14) 

$V = 24\pi \text{ in}^3$

$\pi r^2 h = 24\pi$

$h = \frac{24}{r^2}$

$C = .15\pi r^2 + .05\pi r^2\left(\frac{24}{r^2}\right)$

$C = .15\pi r^2 + \frac{2.4\pi}{r}$

$C' = .3\pi r - \frac{2.4\pi}{r^2} = 0$

$h = \frac{24}{4} = \boxed{6 \text{ in}}$

$\frac{.3\pi r^3 - 2.4\pi}{r^2} = 0$

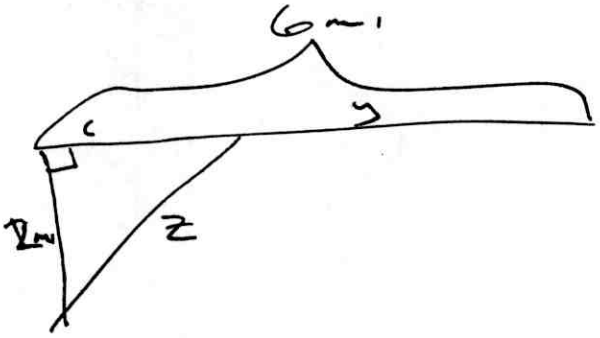
$.3\pi r^3 = 2.4\pi$

$r^3 = 8$

$\boxed{r = 2 \text{ in}}$

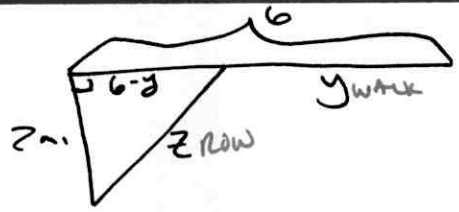
$C = .15\pi(4) + .05\pi(4)(6)$

$C = \$5.65$

15) 

$\frac{y}{3}$

15)



WALK 5 mi/hr

ROW 3 mi/hr

$z^2 = 2^2 + (6-y)^2$

$z = \sqrt{4+(6-y)^2}$

$t = \frac{z}{3} + \frac{y}{5}$

$t = \frac{\sqrt{4+(6-y)^2}}{3} + \frac{y}{5}$

$t' = \frac{-2(6-y)}{6\sqrt{4+(6-y)^2}} + \frac{1}{5} = 0$

$\frac{-12+2y+6\sqrt{4+(6-y)^2}}{30\sqrt{4+(6-y)^2}} = 0$

$-60+10y+6\sqrt{4+(6-y)^2} = 0$

$6\sqrt{4+(6-y)^2} = 60-10y$

$\sqrt{4+(6-y)^2} = 10-\frac{5}{3}y$

$4+(6-y)^2 = 100-\frac{100}{3}y+\frac{25}{9}y^2$

$y^2-12y+40 = 100-\frac{100}{3}y+\frac{25}{9}y^2$

$\frac{16}{9}y^2-\frac{64}{3}y+60 = 0$

$y = 4.5$

6-y

1.5

z z

$z^2 = 4+1.5^2$

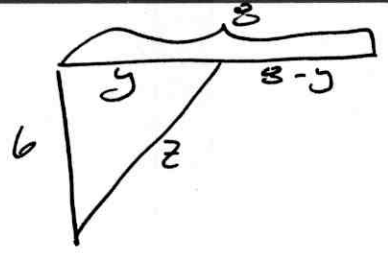
$z^2 = 6.25$

$z = 2.5$

$t = \frac{2.5}{3} + \frac{4.5}{5}$

$t = 1.733$ HOURS

16)



$z = \sqrt{36+y^2}$

$COST = z(100{,}000) + (8-y)(75000)$

$COST = \sqrt{36+y^2}(100000) + 600000 - 75000y$

$C' = \frac{50000(2y)}{\sqrt{36+y^2}} - 75000 = 0$

$C' = \frac{100{,}000y - 75000\sqrt{36+y^2}}{\sqrt{36+y^2}} = 0$

$100000y - 75000\sqrt{36+y^2} = 0$

$100000y = 75000\sqrt{36+y^2}$

$\frac{4}{3}y = \sqrt{36+y^2}$

$\frac{16}{9}y^2 = 36+y^2$

$\frac{7}{9}y^2 = 36$

$y^2 = \frac{324}{7}$

$y = \sqrt{\frac{324}{7}} \approx$ **6.803 miles**

$COST = \sqrt{36+6.803^2}(100000) + (8-6.803)(75000)$

$COST = 907087.697 + 89775$

COST = $996,862.70