AP Calculus AB

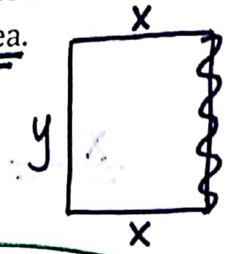
3.7 Optimization Day 1

Name:________________________

Strategies for Solving Optimization Problems:

- Read the problem carefully. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?
- Draw a diagram.
- Introduce variables. Write a primary equation for the quantity to be optimized.
- Reduce the primary equation to one having a single independent variable.
- Solve and consider domain.

1) A rectangular garden is to be enclosed using the wall of a building as one side and 60 feet of fencing on the other three sides. Find the length and width that will give the maximum area.



$2x+y=60$

$A=xy$

$A=x(60-2x)$

$A=60x-2x^2$

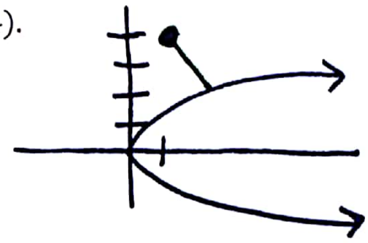
$\frac{dA}{dx}=60-4x=0$

$x=15$

$y=30$

15 ft by 30 ft

2) Find the point on the parabola $y^2=2x$ that is closest to the point (1, 4).



$y^2=2x$

$x=\frac{1}{2}y^2$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(x-1)^2+(y-4)^2}$

$d=\sqrt{(\frac{1}{2}y^2-1)^2+(y-4)^2}$

$\downarrow$

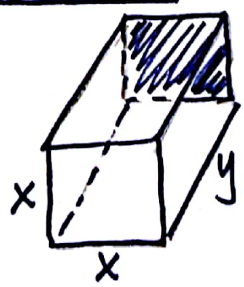
$d=(\frac{1}{2}y^2-1)^2+(y-4)^2$

$\frac{dd}{dy}=2(\frac{1}{2}y^2-1)'\cdot y+2(y-4)'\cdot 1=0$

$y^3-8=0 \quad y^3=8 \quad y=2$

(2, 2)

3) A box in the shape of a rectangular prism has one square base and the other square base is open. It is made with 48 square feet of material. What dimensions will result in a box with the largest volume?



$48=x^2+4xy$

$\frac{48-x^2}{4x}=\frac{4xy}{4x}$

$y=\frac{48-x^2}{4x}$

$y=\frac{48-4^2}{4(4)}$

$V=x^2y$

$V=x^2\left(\frac{48-x^2}{4x}\right)=\frac{1}{4}x(48-x^2)$

$12x-\frac{1}{4}x^3$

$V=\frac{48x^2-x^4}{4x}$

$\frac{dV}{dx}=12-\frac{3}{4}x^2=0$

$x^2=16$

$x=\pm 4$

$x=4$ ft

$y=2$ ft

u Try:

4) Find two non-negative numbers whose sum is 9 and so that the product of one number and the square of the second number is a maximum.

$x+y=9$

$x=9-y$

$x\cdot y^2=M$

$(9-y)y^2=M$

$9y^2-y^3=M$

$\frac{dM}{dy}=18y-3y^2=0$

$3y(6-y)=0$

$\boxed{y=6,\ \cancel{0} \quad x=3}$

5) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y=9-x^2$.

$y=9-x^2$

$A=2x\cdot y$

$A=2x(9-x^2)$

$A=18x-2x^3$

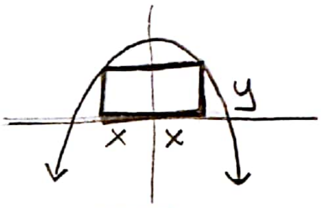
$\frac{dA}{dx}=18-6x^2=0$

$x^2=3$

$x=\pm\sqrt{3}$

$y=9-(\sqrt{3})^2$

$y=9-3=6$



$\boxed{2x=\text{length}=2\sqrt{3} \quad y=\text{width}=6}$

6) You want to build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?

$8x+5y=500$

$8x=500-5y$

$x=\frac{500-5y}{8}$

$A=4x\cdot y$

$A=4\left(\frac{500-5y}{8}\right)\cdot y$

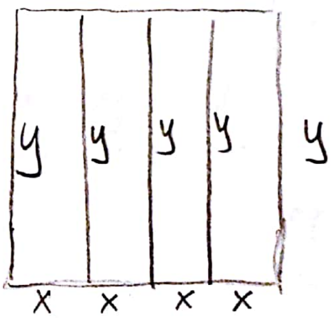
$A=\left(\frac{2000-20y}{8}\right)y$

$A=\frac{2000y-20y^2}{8}$

$A=\frac{1}{8}(2000y-20y^2)$

$\frac{dA}{dy}=\frac{1}{8}(2000-40y)=0$

$\boxed{125\text{ ft by }50\text{ ft}}$



$2000=40y$

$y=50$

$x=\frac{500-5(50)}{8}$

$x=\frac{250}{8}=31.25$

$=\frac{125}{4}$

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$$2x + 5y = 500$$
$$2x = 500 - 5y$$
$$x = \frac{500 - 5y}{2}$$
$$x = 250 - \tfrac{5}{2}y$$

$$A = x \cdot y$$
$$A = y\left(250 - \tfrac{5}{2}y\right)$$
$$A = 250y - \tfrac{5}{2}y^2$$
$$\frac{dA}{dy} = 250 - 5y = 0$$
$$-5y = -250$$
$$y = 50$$

$$x = \frac{500 - 5(50)}{2}$$
$$x = 125$$