

Optimization WS:

1) $x + y = 6$

$y = 6 - x$

$x^2 + y^2 = M$

$x^2 + (6-x)^2 = M$

$\frac{dM}{dx} = 2x + 2(6-x) \cdot -1$

$\frac{dM}{dx} = 2x - 12 + 2x = 0$

$4x - 12 = 0$

$4x = 12$

$x = 3$
 $y = 3$

2) $xy = 16$

$x = \frac{16}{y}$

$x + y = M$

$\frac{16}{y} + y = M$

$M = 16y^{-1} + y$

$\frac{dM}{dy} = -16y^{-2} + 1$

$-\frac{16}{y^2} + \frac{y^2}{y^2} = 0$

$-16 + y^2 = 0$

$y^2 = 16$
 $y = \pm 4$

$y = 4$
 $x = 4$

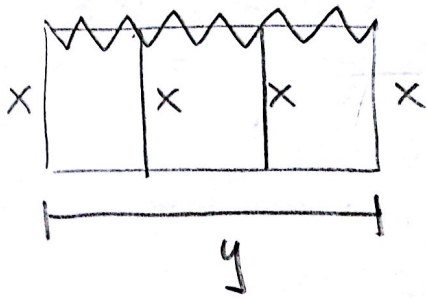
3) $x + 4\left(\frac{1}{x}\right)^2 = M$

$M = x + 4x^{-2}$

$\frac{dM}{dx} = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3} = 0$

$x^3 - 8 = 0$
 $x = 2$

4) $4x + y = 200$
 $4y = 200 - 4x$



$$A = xy$$

$$A = x(200 - 4x)$$

$$A = 200x - 4x^2$$

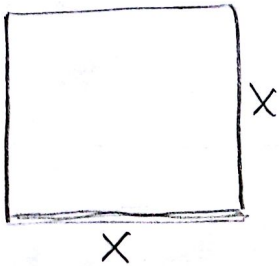
$$\frac{dA}{dx} = 200 - 8x = 0$$

$$x = 25$$

$$y = 100$$

$A = 2500 \text{ ft}^2$

5)



$$V = 500$$

$$500 = x \cdot x \cdot h$$

$$500 = x^2 h$$

$$h = \frac{500}{x^2}$$

$$SA = x^2 + 4xh$$

$$SA = x^2 + 4x \left(\frac{500}{x^2} \right)$$

$$SA = x^2 + 2000x^{-1}$$

$$\frac{dSA}{dx} = 2x - \frac{2000}{x^2}$$

$$= \frac{2x^3 - 2000}{x^2} = 0$$

$$2x^3 = 2000$$

$$x^3 = 1000$$

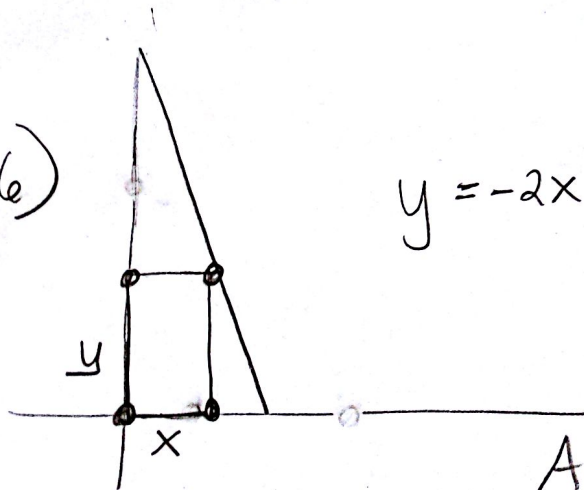
10 by 10 by 5

$$x = 10$$

$$h = \frac{500}{100} = 5$$

6)

$$y = -2x + 100$$



$$A = xy$$

$$A = x(-2x + 100)$$

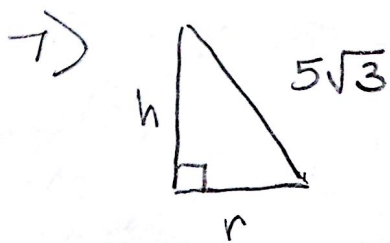
$$A = -2x^2 + 100x$$

$$\frac{dA}{dx} = -4x + 100 = 0$$

$$x = 25$$

$$y = 50$$

1250 units^2



$$\begin{aligned} r^2 + h^2 &= (5\sqrt{3})^2 \\ r^2 + h^2 &= 75 \\ r^2 &= 75 - h^2 \end{aligned}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (75 - h^2) \cdot h$$

$$V = \frac{1}{3} \pi (75h - h^3)$$

$$\frac{dV}{dh} = \frac{1}{3} \pi (75 - 3h^2)$$

$$V = \frac{1}{3} \pi r^2 (5)$$

$$V = \frac{1}{3} \pi (5\sqrt{2})^2 \cdot 5$$

$$V = \frac{1}{3} \pi (250)$$

$$V = \frac{250\pi}{3} \text{ in}^3$$

$$0 = \frac{1}{3} \pi (75 - 3h^2)$$

$$75 - 3h^2 = 0$$

$$-3h^2 = -75$$

$$h^2 = 25$$

$$h = 5$$

$$r^2 = 75 - (5)^2$$

$$r^2 = 50$$

$$r = \sqrt{50} = 5\sqrt{2}$$

8) $x + y = 10$

$$x = 10 - y$$

$$x^3 + y^3 = M$$

$$(10 - y)^3 + y^3 = M$$

$$\frac{dM}{dy} = 3(10 - y)^2 \cdot (-1) + 3y^2$$

$$\frac{dM}{dy} = -3(10 - y)^2 + 3y^2$$

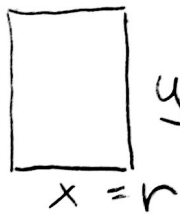
$$0 = -3(100 - 20y + y^2) + 3y^2$$

$$0 = -300 + 60y - 3y^2 + 3y^2$$

$$300 = 60y$$

$$\begin{aligned} y &= 5 \\ x &= 5 \end{aligned}$$

9)



$$y = h$$

$$2x + 2y = 36$$

$$2r + 2h = 36$$

$$2h = 36 - 2r$$

$$h = 18 - r$$

$$h = 18 - 12$$

$$h = 6$$

$$V = \pi(12)^2 \cdot 6$$

$$V = 864\pi \text{ in}^3$$

$$V = \pi r^2 h$$

$$V = \pi r^2 (18 - r)$$

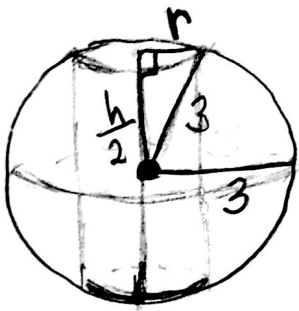
$$V = \pi(18r^2 - r^3)$$

$$\frac{dV}{dr} = \pi(36r - 3r^2)$$

$$0 = 3\pi r(12 - r)$$

$$r = 0, 12$$

10)



$$r^2 + \left(\frac{h}{2}\right)^2 = 9$$

$$r^2 = 9 - \left(\frac{h}{2}\right)^2$$

$$r^2 = 9 - \frac{1}{4}h^2$$

$$V_{\text{cyl}} = \pi r^2 h$$

$$V = \pi\left(9 - \frac{1}{4}h^2\right)h$$

$$V = \pi\left(9h - \frac{1}{4}h^3\right)$$

$$\frac{dV}{dh} = \pi\left(9 - \frac{3}{4}h^2\right)$$

$$9 - \frac{3}{4}h^2 = 0$$

$$-\frac{3}{4}h^2 = -9$$

$$\left(\frac{4}{3}\right)\frac{3}{4}h^2 = 9\left(\frac{4}{3}\right)$$

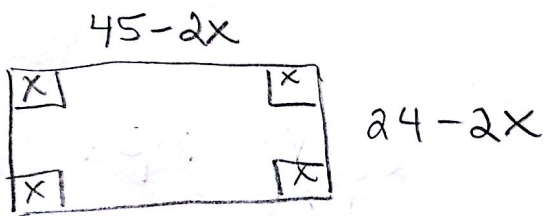
$$h^2 = 12$$

$$h = 2\sqrt{3}$$

$$V = \pi(6)(2\sqrt{3})$$

$$V = 12\pi\sqrt{3} \text{ in}^3$$

11)



$$V = x(24-2x)(45-2x)$$

$$V = x(1080 - 138x + 4x^2)$$

$$V = 1080x - 138x^2 + 4x^3$$

$$\frac{dV}{dx} = 1080 - 276x + 12x^2 = 0$$

$$12(x^2 - 23x + 90) = 0$$

$$12(x-18)(x-5) = 0$$

$$x = 5:$$

$$V = (35)(14)(5)$$

$$2450 \text{ in}^3$$

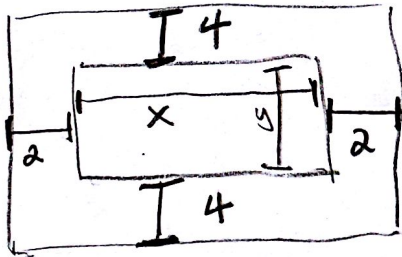
$$x = 18, 5$$

~~$$x = 18:$$~~

~~$$V = (9)(-12)$$~~

35 by 14 by 5 in

12)



$$x = 4$$

$$A = 50 \text{ in}^2 \quad A = (x+4)(y+8)$$

$$50 = xy \quad A = \left(\frac{50}{y} + 4\right)(y+8)$$

$$x = \frac{50}{y}$$

$$A = 50 + \frac{400}{y} + 4y + 32$$

$$A = 82 + 4y + 400y^{-1}$$

$$\frac{dA}{dy} = 4 - \frac{400}{y^2} = 0$$

$$4y^2 = 400$$

$$y^2 = 100$$

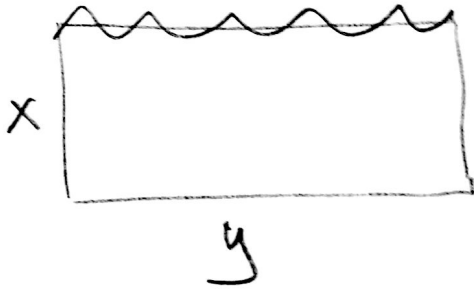
$$y = 10$$

$$x = 5$$

$$\frac{4y^2 - 400}{y^2} = 0$$

9 by 18

13)



$$A = 200$$

$$P = 2x + y$$

$$200 = xy$$

$$P = 2\left(\frac{200}{y}\right) + y$$

$$x = \frac{200}{y}$$

$$P = 400y^{-1} + y$$

$$\frac{dP}{dy} = -\frac{400}{y^2} + 1$$

$$-400 + y^2 = 0$$

$$y^2 = 400$$

$$y = 20$$

$$x = 10$$

10 by 20