*AP Calculus AB*

Optimization Worksheet, Section 3.7

1. The sum of two numbers is 6. The sum of their squares is a minimum. What are the two numbers?

2. The product of two positive numbers is 16. Their sum is a minimum. What are the two numbers?

3. The sum of a positive number and 4 times the square of its reciprocal is a minimum. What is the number?

4. A rectangular piece of land borders a wall. The land is to be enclosed and divided by 200 feet of fencing, as shown below. (No fence is along the wall). What is the largest area that can be enclosed?
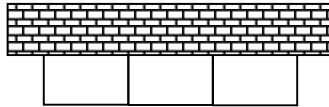
5. An open rectangular box has a square base and a volume of 500 cubic inches. What dimensions minimize the amount of cardboard needed to make the box?

6. A rectangle with horizontal and vertical sides has one vertex at the origin, one on the positive x-axis, one on the positive y-axis, and one on the line $2x + y = 100$. What is the maximum possible area of this rectangle?

7. A right triangle with hypotenuse of $5\sqrt{3}$ inches is revolved about one of its legs to generate a cone. What is volume of the cone of the greatest possible volume?

8. The sum of two numbers is 10. The sum of their cubes is a minimum. What are the two numbers?

9. A rectangle with perimeter of 36 inches is revolved about one of its sides to form a cylinder. What is the maximum possible volume that could be generated?

10. A cylinder is inscribed in a sphere with radius 3 inches. What is the volume of the cylinder with the greatest possible volume?

11. An open rectangular box is formed by cutting congruent squares from the corners of a piece of cardboard and folding the sides up. If the original piece of cardboard was 24 inches by 45 inches, what are the dimensions of the box with maximum volume?

12. A poster is to contain 50 square inches of print. The top an bottom margins are 4 inches each, and the left and right margins are 2 inches each. What dimensions minimize the area of the poster?

13. A rectangular piece of land has an area of 200 square feet, and is enclosed by fencing on 3 sides. (The fourth side is along a river). What dimensions minimize the amount of fence needed?

Answers:

1.   3, and 3

2.   4, and 4

3.   2

4.   2500 square feet

5.   10 inches, by 10 inches, by 5 inches

6.   1250 square units

7. $\frac{250\pi}{3}$ cubic inches

8.   5, and 5

9. 864π cubic inches

10. $12\pi\sqrt{3}$ cubic inches

11.   35 inches, by 14 inches, by 5 inches

12.   9 feet high, by 18 feet high

13.   10 feet by 20 feet