Day 8 – Optimization

Key

Objective: Use derivatives to solve problems involving optimization

Optimization involves finding the largest or smallest values of a function. As with all functions, the domain is vital to determining the solution

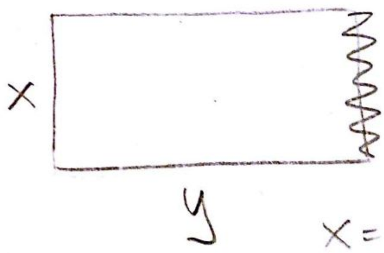
Steps in Solving Optimization Problems

- Understand the problem. Read the problem carefully. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?
- Draw a Diagram.
- Introduce variables. Write a primary equation for the quantity to be optimized
- Reduce the primary equation to one having a single independent variable.
- Determine the feasible domain.
- Solve

Example #1

A rectangular garden is to be enclosed using the wall of a building as one side and 60-feet of fencing on the other three sides. Find the length and width that will give the maximum area.

$\boxed{\begin{array}{l} l = 30 \\ w = 15 \end{array}}$



$P = 2y + x$

$60 = x + 2y$

$x = 60 - 2y$

$A = xy$

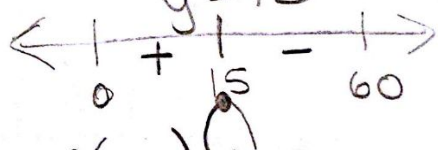
$A = (60 - 2y)y$

$A = 60y - 2y^2$

$\frac{dA}{dy} = 60 - 4y = 0$

$-4y = -60$

$y = 15$



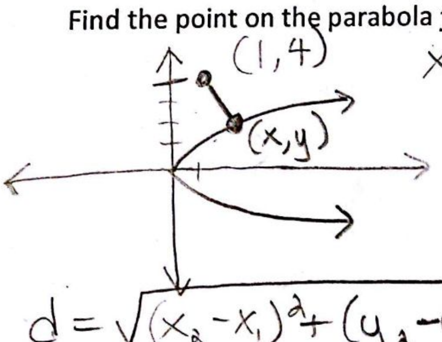
$x = 60 - 2(15)$

$x = 30$

$A = 15 \cdot 30 = 450 \text{ ft}^2$

Example 2

Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4).



$x = \frac{1}{2}y^2 \qquad y = \pm\sqrt{2x}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-1)^2 + (y-4)^2}$

$d = \sqrt{\left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2}$

$d = \left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2$

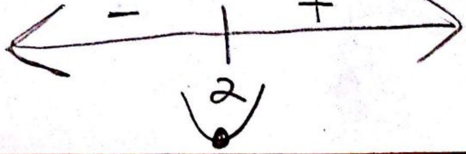
$\frac{dd}{dy} = 2\left(\frac{1}{2}y^2 - 1\right) \cdot y + 2(y-4) \cdot 1$

$\frac{dd}{dy} = (y^2 - 2)y + 2y - 8$

$\frac{dd}{dy} = y^3 - 2y + 2y - 8$

$\frac{dd}{dy} = y^3 - 8 = 0$

$y = 2$

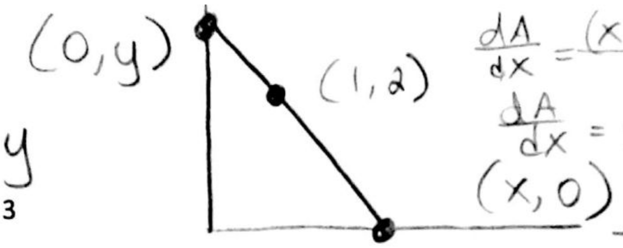


$x = \frac{1}{2}(2)^2$

$x = 2$

$\boxed{(2, 2)}$

$A=\frac{1}{2}xy$



$\frac{dA}{dx}=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$

$\frac{dA}{dx}=\frac{2x^2-2x-x^2}{(x-1)^2}$

$\frac{dA}{dx}=\frac{x^2-2x}{(x-1)^2}=0$

$x(x-2)=0$

$x=0,2$

$-$ $+$

0 2

$x=2$

$y=4$

Example 3

A right triangle is formed in the 1st quadrant by the x- and y- axes and a line through the point (1,2).

a) Write the length of L of the hypotenuse as a function of x.

$L^2=x^2+y^2$

$L^2=x^2+\left(\frac{-2x}{1-x}\right)^2$

(0,0) (0,4) (2,0)

b) Find the vertices of the triangle such that its area is a minimum.

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{2-y}{1-0}=\frac{2-0}{1-x}$

$\frac{2-y}{1}=\frac{2}{1-x}$

$2-y=\frac{2}{1-x}$

$-y=\frac{2}{1-x}-2$

$y=\frac{-2}{1-x}+2\cdot\frac{(1-x)}{(1-x)}$

$y=\frac{\cancel{-2}+\cancel{2}-2x}{(1-x)}$

$y=\frac{-2x}{(1-x)}$

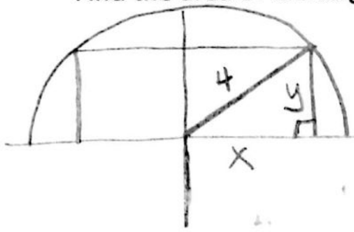
$A=\frac{1}{2}x\left(\frac{-2x}{1-x}\right)$

$A=\frac{-x^2}{1-x}$

$A=\frac{x^2}{x-1}$

Example 4

Find the area of the largest rectangle that can be inscribed in a semicircle of radius 4.



$x^2+y^2=4^2$

$x^2+y^2=16$

$y=\sqrt{16-x^2}$

$A=2xy$

$A=2x\left(\sqrt{16-x^2}\right)$

$\frac{dA}{dx}=2\left[x\left(\frac{1}{2}(16-x^2)^{-1/2}\cdot-2x\right)+\sqrt{16-x^2}\cdot 1\right]$

$\frac{dA}{dx}=2\left[\frac{-x^2}{\sqrt{16-x^2}}+\frac{\sqrt{16-x^2}}{1}\cdot\frac{\sqrt{16-x^2}}{\sqrt{16-x^2}}\right]$

$\frac{dA}{dx}=2\left[\frac{-x^2+16-x^2}{\sqrt{16-x^2}}\right]$

$\frac{2(-2x^2+16)}{\sqrt{16-x^2}}$

$-2x^2+16=0$

$x^2=8$

$x=\sqrt{8}$

$(\sqrt{8})^2+y^2=16$

$8+y^2=16$

$y^2=8$

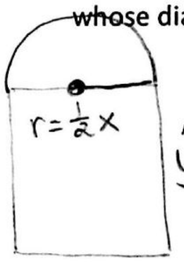
$y=\sqrt{8}$

$A=2(\sqrt{8})(\sqrt{8})$

$=\boxed{16\ u^2}$

Example 5

Suppose that 600 ft of fencing are used to enclose a corral in the shape of a rectangle with a semicircle whose diameter is a side of the rectangle. Find the dimensions of the corral with maximum area.



$A=xy+\frac{1}{2}\pi\left(\frac{1}{2}x\right)^2$

$A=xy+\frac{1}{8}\pi x^2$

$P=600=2y+x+\frac{1}{2}\pi x$

$600-x-\frac{1}{2}\pi x=2y$

$y=\frac{600-x-\frac{1}{2}\pi x}{2}$

$y=300-\frac{1}{2}x-\frac{1}{4}\pi x$

$A=x\left(300-\frac{1}{2}x-\frac{1}{4}\pi x\right)+\frac{1}{8}\pi x^2$

$A=300x-\frac{1}{2}x^2-\frac{1}{4}\pi x^2+\frac{1}{8}\pi x^2$

$\frac{dA}{dt}=300-x-\frac{1}{2}\pi x+\frac{1}{4}\pi x$

$0=300-x-\frac{1}{4}\pi x$

$-300=-x\left(1+\frac{1}{4}\pi\right)$

$300=x\left(1+\frac{1}{4}\pi\right)$

$x=\frac{300}{1+\frac{1}{4}\pi}$ $y=$

Scanned by CamScanner