AP Calculus AB

3.7 Optimization Day 1

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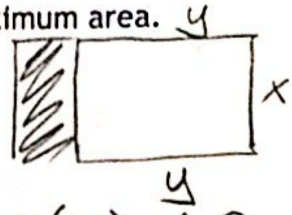
**Strategies for Solving Optimization Problems:**

- Read the problem carefully. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?
- Draw a diagram.
- Introduce variables. Write a primary equation for the quantity to be optimized.
- Reduce the primary equation to one having a single independent variable.
- Determine the feasible domain.
- Solve.

x: (0, 60)
y: (0, 60)

1) A rectangular garden is to be enclosed using the wall of a building as one side and 60 feet of fencing on the other three sides. Find the length and width that will give the maximum area.




$A = xy$

$P = 2y + x = 60$

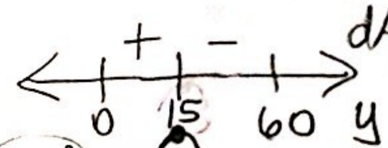
$x = -2y + 60$

$A = (60 - 2y)y$

$A = 60y - 2y^2$

$\frac{dA}{dt} = 60 - 4y = 0$

$y = 15$

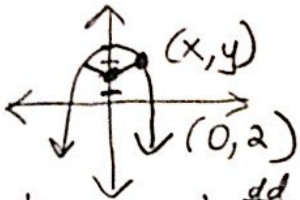



$x = -2(15) + 60$

$\boxed{x = 30 \quad y = 15}$

2) Which points on the graph of y = 4 - $x^2$ are closest to the point (0, 2)?

min distance




$d = \sqrt{(x-0)^2 + (y-2)^2}$

$d = \sqrt{x^2 + (4 - x^2 - 2)^2}$

$d = \sqrt{x^2 + (2 - x^2)^2}$

$d = \sqrt{x^2 + 4 - 4x^2 + x^4}$

$d = \sqrt{x^4 - 3x^2 + 4}$

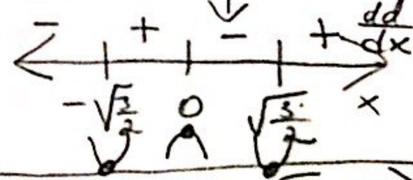
$d = x^4 - 3x^2 + 4$

$\frac{dd}{dx} = 4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

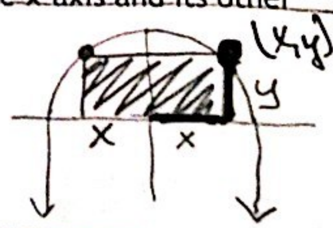
$2x = 0 \qquad 2x^2 = 3$

$x = 0 \qquad x = \pm\sqrt{\frac{3}{2}}$




$\boxed{\left(\sqrt{\tfrac{3}{2}}, \tfrac{5}{2}\right) \left(-\sqrt{\tfrac{3}{2}}, \tfrac{5}{2}\right)}$

3) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the y-axis and lying on the parabola y = 9 - $x^2$.




$A = 2x \cdot y$

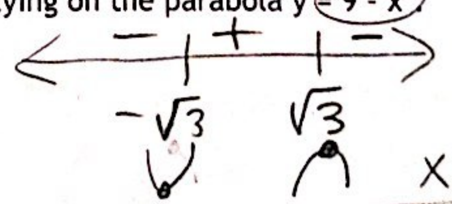
$A = 2x(9 - x^2)$

$A = 18x - 2x^3$

$\frac{dA}{dx} = 18 - 6x^2 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$




$x = \sqrt{3}$

length $= 2\sqrt{3}$

width $= 9 - (\sqrt{3})^2 = 6$