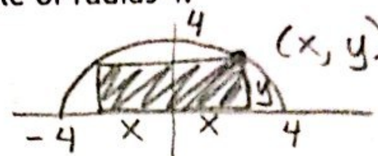
Find the area of the largest rectangle that can be inscribed in a semicircle of radius 4.



$A = 2x \cdot y$

$x^2 + y^2 = 4^2$

$x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \sqrt{16 - x^2}$

$A = 2x\left(\sqrt{16-x^2}\right)$

$A = 2x \cdot (16 - x^2)^{1/2}$

$\frac{dA}{dx} = 2x\left[\frac{1}{2}(16-x^2)^{-1/2} \cdot -2x\right] + (16-x^2)^{1/2} \cdot 2$

$\frac{dA}{dx} = \frac{-2x^2}{\sqrt{16-x^2}} + 2\sqrt{16-x^2} \cdot \frac{\sqrt{16-x^2}}{\sqrt{16-x^2}}$

$\frac{dA}{dx} = \frac{-2x^2 + 2(16-x^2)}{\sqrt{16-x^2}}$

$\frac{dA}{dx} = \frac{-2x^2 + 32 - 2x^2}{\sqrt{16-x^2}}$

$\frac{dA}{dx} = \frac{-4x^2 + 32}{\sqrt{16-x^2}} = 0$

$-4x^2 + 32 = 0$

$-4x^2 = -32$

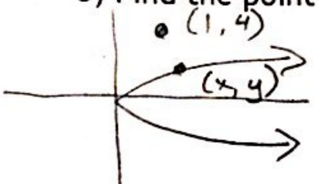
$x^2 = 8$

$x = \sqrt{8}$

$A = 2(\sqrt{8}) \cdot \sqrt{16 - (\sqrt{8})^2}$

$A = 2\sqrt{8} \cdot \sqrt{8} = \boxed{16}$

5) Find the point on $y^2 = 2x$ that is closest to the point (1, 4).



$y = \sqrt{2x} \qquad x = \frac{y^2}{2}$

$d = \sqrt{(x-1)^2 + (y-4)^2}$

$d = \sqrt{\left(\frac{y^2}{2}-1\right)^2 + (y-4)^2}$

$d = \left(\frac{y^2}{2}-1\right)^2 + (y-4)^2$

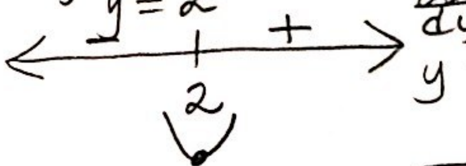
$\frac{dd}{dy} = 2\left(\frac{y^2}{2}-1\right) \cdot y + 2(y-4) \cdot 1$

$\frac{dd}{dy} = 2y\left(\frac{y^2}{2}-1\right) + 2(y-4)$

$\frac{dd}{dy} = y^3 - \cancel{2y} + \cancel{2y} - 8$

$\frac{dd}{dy} = y^3 - 8 = 0$

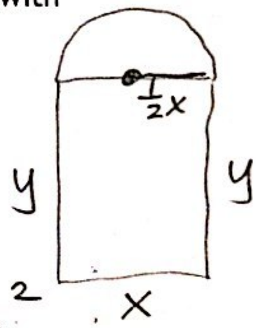
$y = 2$



$y = 2$

$x = \frac{2^2}{2} = 2$ $\boxed{(2, 2)}$

6) Suppose that 600 feet of fencing is used to enclose a corral in the shape of a rectangle with a semicircle whose diameter is a side of the rectangle. Find the dimensions of the corral with maximum area.



$P = 600 = 2y + x + \frac{2\pi\left(\frac{1}{2}x\right)}{2}$

$\frac{600}{2} = \frac{2y + x + \frac{1}{2}\pi x}{2}$

$300 = y + \frac{1}{2}x + \frac{1}{4}\pi x$

$y = 300 - \frac{1}{2}x - \frac{1}{4}\pi x$

$A = x \cdot y + \frac{\pi\left(\frac{1}{2}x\right)^2}{2}$

$A = xy + \frac{1}{4}\pi x^2 \cdot \frac{1}{2}$

$A = xy + \frac{1}{8}\pi x^2$

$A = x\left(300 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{1}{8}\pi x^2$

$A = 300x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x^2$

$A = 300x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$

$\frac{dA}{dx} = 300 - x - \frac{1}{4}\pi x = 0$

$-300 = -x - \frac{1}{4}\pi x$

$-300 = -x\left(1 + \frac{1}{4}\pi\right)$

$$\boxed{x = \frac{300}{\left(1 + \frac{\pi}{4}\right)} \qquad y = P\left(\frac{300}{\left(1 + \frac{\pi}{4}\right)}\right)}$$