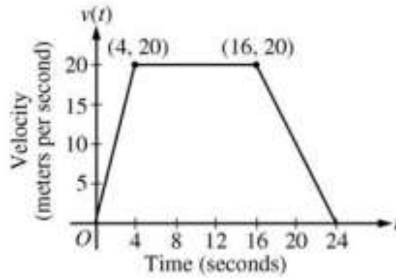
AP® CALCULUS AB
2005 SCORING GUIDELINES

Question 5



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

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Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$$\lim_{t \to 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \to 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

$3 : \begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$

(c)

$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2 : \begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2 : \begin{cases} 1 : \text{average rate of change of } v \text{ on } [8, 20] \\ 1 : \text{answer with explanation} \end{cases}$

(a) Average acceleration of rocket A is

$$\frac{v(80)-v(0)}{80-0}=\frac{49-5}{80}=\frac{11}{20}\ \text{ft}/\text{sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is

$$20[v(20)+v(40)+v(60)]$$
$$= 20[22+35+44] = 2020 \text{ ft}$$

3 :
- 1 : explanation
- 1 : uses $v(20)$, $v(40)$, $v(60)$
- 1 : value

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$$v_B(t)=\int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1}+C$$
$$2 = v_B(0) = 6 + C$$
$$v_B(t) = 6\sqrt{t+1}-4$$
$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

4 :
- 1 : $6\sqrt{t+1}$
- 1 : constant of integration
- 1 : uses initial condition
- 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion

Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)