

Day 3 - More related rates

Objective: Use basic math formulas and derivatives to solve problems

1. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep.

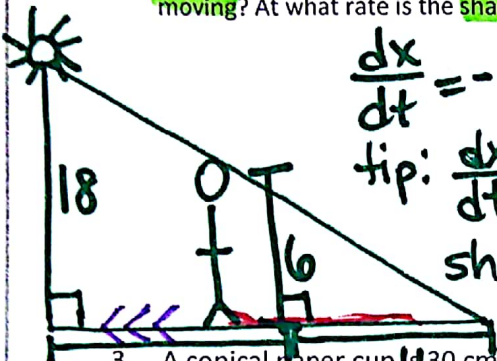


$r = 2$
 $h = 4$
 $r = \frac{1}{2}h$
 $\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$
 $\frac{dh}{dt} = ?$
 when $h = 3$

$V = \frac{1}{3} \pi r^2 h$
 $V = \frac{1}{3} \pi (\frac{1}{2}h)^2 \cdot h$
 $V = \frac{1}{3} \pi (\frac{1}{4}h^2) \cdot h$
 $V = \frac{1}{12} \pi h^3$

$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$
 $2 = \frac{1}{4} \pi (3)^2 (\frac{dh}{dt})$
 $8 = 9\pi \frac{dh}{dt}$
 $\frac{8}{9\pi} = \frac{dh}{dt}$
 m/min

2. A man 6ft tall is walking at the rate of 3 ft/s toward a streetlight 18 ft high. At what rate is the tip of the shadow moving? At what rate is the shadow length changing?

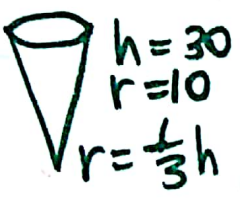


$\frac{dx}{dt} = -3 \text{ ft/s}$
 tip: $\frac{dx}{dt} + \frac{dy}{dt} = ?$
 shadow: $\frac{dy}{dt} = ?$

$\frac{x+y}{18} = \frac{y}{6}$
 $6x + 6y = 18y$
 $x + y = 3y$
 $x = 2y$

$\frac{dx}{dt} = 2 \frac{dy}{dt}$
 $-3 = 2 \frac{dy}{dt}$
 $\frac{dy}{dt} = -\frac{3}{2} \text{ ft/s}$
 $-\frac{3}{2} + -\frac{3}{2} = -\frac{9}{2} \text{ ft/s}$
 SHADOW TIP

3. A conical paper cup is 30 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $2\pi^3$ cm³/sec. How fast is the water level rising when the water level is 2 cm?

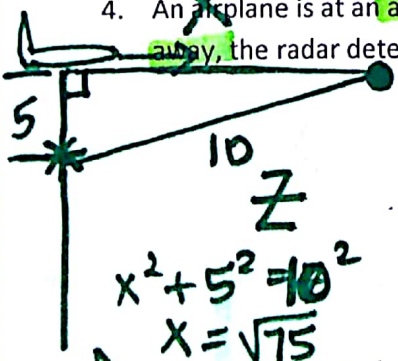


$h = 30$
 $r = 10$
 $r = \frac{1}{3}h$
 $\frac{dV}{dt} = \frac{2\pi^3}{3}$
 $\frac{dh}{dt} = ?$
 when $h = 2$

$V = \frac{1}{3} \pi r^2 h$
 $V = \frac{1}{3} \pi (\frac{1}{3}h)^2 \cdot h$
 $V = \frac{1}{27} \pi h^3$

$\frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt}$
 $\frac{2\pi^3}{3} = \frac{1}{9} \pi (2)^2 \cdot \frac{dh}{dt}$
 $\frac{dh}{dt} = \frac{18\pi^3}{12\pi} = \frac{3}{2}$
 cm/sec

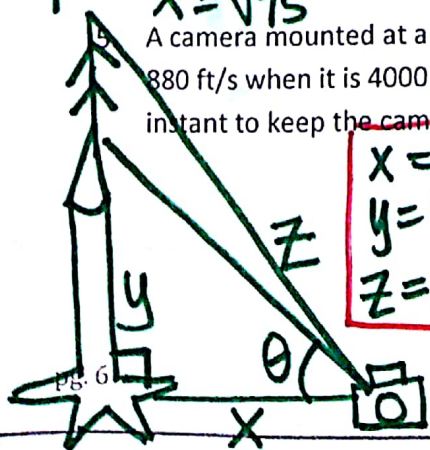
4. An airplane is at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance s is changing at a rate of 240 mph. What is the speed of the airplane?



$\frac{dz}{dt} = 240 \text{ mi/hr}$
 $x^2 + 5^2 = z^2$
 $x = \sqrt{z^2 - 25}$
 $\frac{dx}{dt} = ?$

$x^2 + y^2 = z^2$
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 $2\sqrt{75} \frac{dx}{dt} + 0 = 2(10)(240)$
 $\frac{dx}{dt} = \frac{2400}{\sqrt{75}} \text{ mi/hr}$

5. A camera mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket?



$x = 3000$
 $y = 4000$
 $z = 5000$

$\frac{dy}{dt} = 880 \text{ ft/s}$
 $\frac{d\theta}{dt} = ?$

$\tan \theta = \frac{y}{x}$
 $\tan \theta = \frac{4000}{3000}$
 $\theta = .927$
 $\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{3000} \frac{dy}{dt}$
 $\sec^2(.927) (\frac{d\theta}{dt}) = \frac{1}{3000} (880)$
 $.106 \text{ rad/s}$