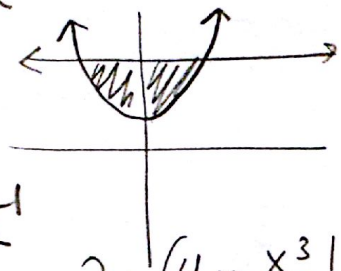
Key

$2\cdot\int_0^2 (5-(x^2+1))\,dx$

$2\cdot\int_0^2 (4-x^2)\,dx$



$5 = x^2+1$

$4 = x^2$

$2\cdot\left(4x-\frac{x^3}{3}\Big|_0^2\right)$

$2\left(8-\frac{8}{3}\right)$

$2\left(\frac{16}{3}\right) = \frac{32}{3}$

Area - Volume Problems

Multiple Choice
Identify the choice that best completes the statement or answers the question.

Underlined questions are calculator required; others are no calculator.

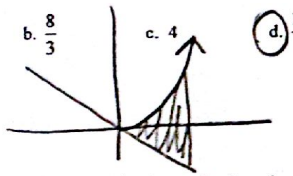
Area: 1, 2, 3, 4, 5
Volume of revolution: 6,
Volume by slicing: 7, 8, 9, 10, 11

1. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

D a. $\frac{14}{3}$ b. $\frac{16}{3}$ c. $\frac{28}{3}$ (d.) $\frac{32}{3}$ e. 8π

2. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

D a. $\frac{2}{3}$ b. $\frac{8}{3}$ c. 4 (d.) $\frac{14}{3}$ e. $\frac{16}{3}$



$\int_0^2 (x^2+x)\,dx$

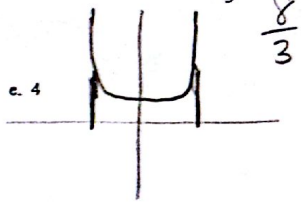
$\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^2$

$\frac{8}{3}+\frac{6}{3}$

$\frac{14}{3}$

* 3. Let R be the region enclosed by the graph of $y = 1+\ln\left(\cos^4 x\right)$ the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

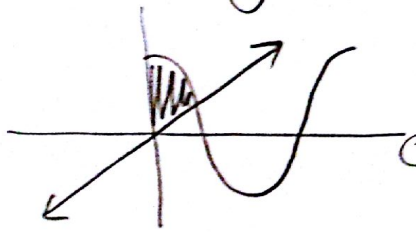
B a. 0 (b.) 1 c. 2 d. 3 e. 4



$\int_{-2/3}^{2/3} 1+\ln(\cos^4 x)$

* 4. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

C a. 0.127 b. 0.385 (c.) 0.400 d. 0.600 e. 0.947



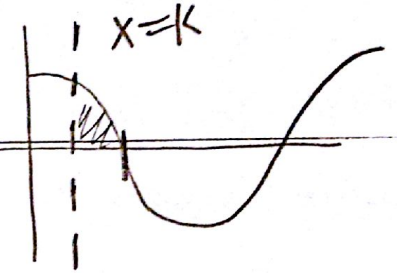
$\cos x = x$

$x = .7391$

$\int_0^{.7391} (\cos x - x)\,dx$

$\sin x\Big|_k^{\pi/2}$

$1-\sin k = .1$



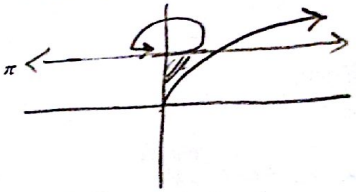
* 5. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

D a. 1.471 b. 1.414 c. 1.277 (d.) 1.120 e. 0.436

$\int_k^{\pi/2} \cos x\,dx = .1$

6. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

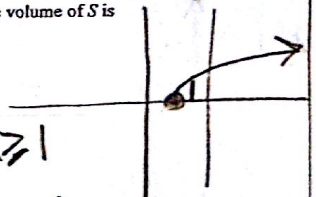
A (a.) $\frac{32\pi}{5}$ b. $\frac{16\pi}{3}$ c. $\frac{16\pi}{5}$ d. $\frac{8\pi}{3}$ e. π



$y^2 = x$

* 7. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

C a. $\frac{1}{2}$ b. $\frac{2}{3}$ (c.) 1 d. 2 e. $\frac{1}{3}\left(e^3-1\right)$



$\int_1^e \left(\sqrt{\ln x}\right)^2 dx$

$x \ge 1$

8. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2-x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

B a. $\pi\int_0^1 (2-y)^2\,dy$
(b.) $\int_0^2 (2-y)\,dy$
c. $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$
d. $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$
e. $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

$y-2 = -x^2$

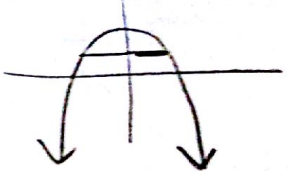
$2-y = x^2$

$\pm\sqrt{2-y} = x$

$\pi\int_0^2 (y^2)^2\,dy$

$\int_1^e \left(\sqrt{\ln x}\right)^2$

$\int \left(\sqrt{2-y}\right)^2$



$\frac{y^5}{5}\Big|_0^2$

$\frac{32}{5}$

$2x - x^2 = 0$

$x^2 - 2x = 0$ $\quad x(x-2) = 0$

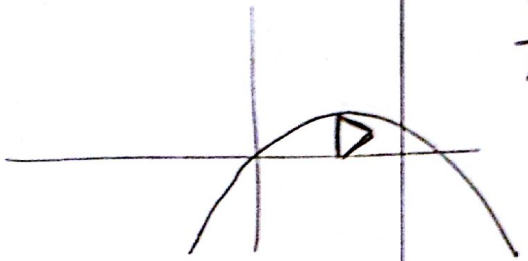
$x = 0, 2$

$$\frac{\sqrt{3}}{4}\int_0^2 (2x - x^2)^2\,dx$$

9. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

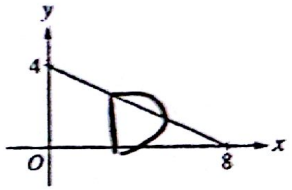
D

a. 1.333
b. 1.067
c. 0.577
d. 0.462
e. 0.267



10.

C



$d = -\frac{1}{2}x + 4$

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

$r = \dfrac{-\frac{1}{2}x + 4}{2}$

a. 12.566
b. 14.661
c. 16.755
d. 67.021
e. 134.041

$x = 8 - 2y$

$\dfrac{2y}{2} = \dfrac{-x+8}{2}$

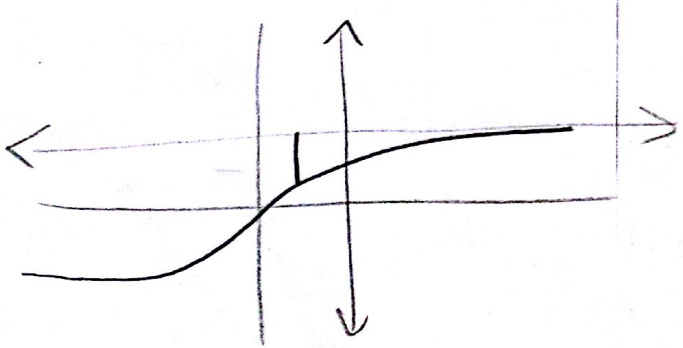
$y = -\frac{1}{2}x + 4$

$$\frac{1}{2}\pi\int_0^8 \left(\frac{-\frac{1}{2}x+4}{2}\right)^2 dx$$

11. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1}x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

B

a. 2.561
b. 5.612
c. 8.046
d. 8.755
e. 20.773



$$\int_0^1 (3 - \tan^{-1}x)^2\,dx$$