The function $f$ is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n$$

for all real numbers $x$ for which the series converges.

(a) Find the interval of convergence of the power series for $f$. Justify your answer.

(b) The power series above is the Taylor series for $f$ about $x = -1$. Find the sum of the series for $f$.

(c) Let $g$ be the function defined by $g(x) = \int_{-1}^{x} f(t)\, dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.

(d) Let $h$ be the function defined by $h(x) = f\left(x^2 - 1\right)$. Find the first three nonzero terms and the general term of the Taylor series for $h$ about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.


The function $f$ is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n$$

for all real numbers $x$ for which the series converges.

(a) Find the interval of convergence of the power series for $f$. Justify your answer.

(b) The power series above is the Taylor series for $f$ about $x = -1$. Find the sum of the series for $f$.

(c) Let $g$ be the function defined by $g(x) = \int_{-1}^{x} f(t)\, dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.

(d) Let $h$ be the function defined by $h(x) = f\left(x^2 - 1\right)$. Find the first three nonzero terms and the general term of the Taylor series for $h$ about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

The function $f$ is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n$$

for all real numbers $x$ for which the series converges.

(a) Find the interval of convergence of the power series for $f$. Justify your answer.

(b) The power series above is the Taylor series for $f$ about $x = -1$. Find the sum of the series for $f$.

(c) Let $g$ be the function defined by $g(x) = \int_{-1}^{x} f(t)\, dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.

(d) Let $h$ be the function defined by $h(x) = f\left(x^2 - 1\right)$. Find the first three nonzero terms and the general term of the Taylor series for $h$ about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

(a) The power series is geometric with ratio $(x+1)$.
The series converges if and only if $|x+1|<1$.
Therefore, the interval of convergence is $-2<x<0$.

$3: \begin{cases} 1: \text{identifies as geometric} \\ 1: |x+1|<1 \\ 1: \text{interval of convergence} \end{cases}$

OR

$$\lim_{n\to\infty}\left|\frac{(x+1)^{n+1}}{(x+1)^n}\right| = |x+1| < 1 \text{ when } -2<x<0$$

At $x=-2$, the series is $\sum_{n=0}^{\infty}(-1)^n$, which diverges since the terms do not converge to 0. At $x=0$, the series is $\sum_{n=0}^{\infty}1$, which similarly diverges. Therefore, the interval of convergence is $-2<x<0$.

OR

$3: \begin{cases} 1: \text{sets up limit of ratio} \\ 1: \text{radius of convergence} \\ 1: \text{interval of convergence} \end{cases}$

(b) Since the series is geometric,

$$f(x) = \sum_{n=0}^{\infty}(x+1)^n = \frac{1}{1-(x+1)} = -\frac{1}{x} \text{ for } -2<x<0.$$

1 : answer

(c) $g\left(-\frac{1}{2}\right) = \int_{-1}^{-\frac{1}{2}} -\frac{1}{x}\,dx = -\ln|x|\Big|_{x=-1}^{x=-\frac{1}{2}} = \ln 2$

$2: \begin{cases} 1: \text{antiderivative} \\ 1: \text{value} \end{cases}$

(d) $h(x) = f\left(x^2-1\right) = 1 + x^2 + x^4 + \cdots + x^{2n} + \cdots$

$h\left(\frac{1}{2}\right) = f\left(-\frac{3}{4}\right) = \frac{4}{3}$

$3: \begin{cases} 1: \text{first three terms} \\ 1: \text{general term} \\ 1: \text{value of } h\left(\frac{1}{2}\right) \end{cases}$