Mini Exam 2:

1) $y = 3x^5 - 5x^2 + 6$

A $\frac{dy}{dx} = 15x^4 - 10x$

$dy = (15x^4 - 10x)dx$

2) $y = x^3 - 3x^2 + x - 1$

B $y' = 3x^2 - 6x + 1$

$y'(2) = 3(2)^2 - 6(2) + 1$

$y'(2) = 12 - 12 + 1 = 1$

3) $x^3 + y^3 = 3xy$

C $3x^2 + 3y^2 \frac{dy}{dx} = 3x \cdot \frac{dy}{dx} + y \cdot 3$

$\frac{dy}{dx}(3y^2 - 3x) = 3y - 3x^2$

$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$

$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$

4) $\lim_{x \to \infty} \left(\frac{7x^2 + 5x - 3}{2 + 3x - 11x^2} \right)$

C $= -\frac{7}{11}$

• 5) $\frac{dV}{dt} = 48\pi \text{ ft}^3/\text{min}$

D $d = 40$ ft, $r = 20$ ft

$h = 60$ ft

$\frac{dh}{dt} = ?$ when $h = 48$ ft

$\frac{r}{h} = \frac{20}{60}$

$20h = 60r$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{3}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi \left(\frac{1}{9}h^2\right) \cdot h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$48\pi = \frac{1}{9}\pi (48)^2 \cdot \frac{dh}{dt}$

$\frac{1}{48} = \frac{1}{9}\frac{dh}{dt}$

$\frac{9}{48} = \frac{dh}{dt}$

$\frac{3}{16}$ ft/min

6) $f(x) = \tan(e^{\sin x})$

B $f'(x) = \sec^2(e^{\sin x}) \cdot e^{\sin x} \cdot \cos x$

7) $y = \sqrt{8 - x^2}$

D $y = (8 - x^2)^{1/2}$

$y' = \frac{1}{2}(8 - x^2)^{-1/2} \cdot -2x$

$y' = \frac{-x}{\sqrt{8 - x^2}}$

$y'(2) = \frac{-2}{\sqrt{8 - 4}} = -\frac{2}{2} = -1$

normal slope $= 1$

8) $y = 4x^8$

E $y' = 32x^7$

$y'\left(\frac{1}{2}\right) = 32\left(\frac{1}{2}\right)^7$

$= 32 \cdot \frac{1}{128}$

$\frac{32}{128} = \frac{1}{4}$

9) $y = e^{3x} \cdot \sin(x^2)$

E $y' = e^{3x} \cdot \cos(x^2) \cdot 2x + \sin(x^2) \cdot 3e^{3x}$