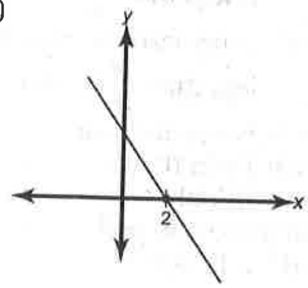
AP CALCULUS BC PRACTICE TEST 1
Section I, Part A: Multiple-Choice Questions
Time: 55 minutes
Number of Questions: 28

A calculator may not be used on this part of the examination.

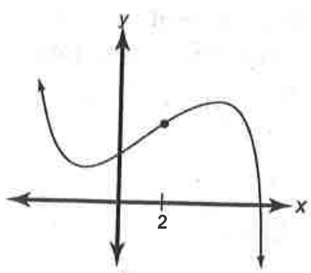
1. If $f(x) = 2x\cos x$, then $f'(x) =$
   (A) $-2\sin x$
   (B) $2x\sin x + 2\cos x$
   (C) $2x\sin x - 2\cos x$
   (D) $-2x\sin x$
   (E) $-2x\sin x + 2\cos x$

2. If $f(x)$ is a function such that $f'(x)$ is increasing for $x < 2$ and $f'(x)$ is decreasing for $x > 2$, then which of the following could be the graph of $f(x)$?

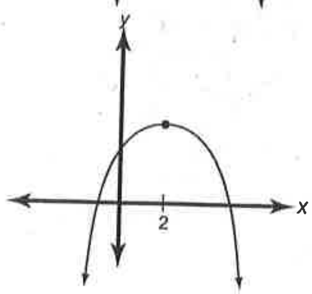
(A)



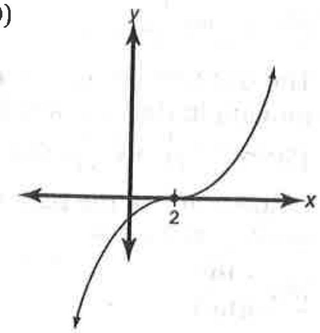
(B)



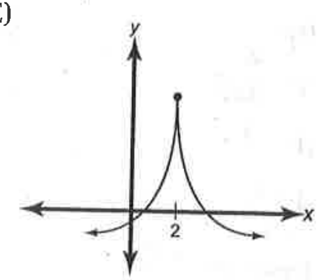
(C)



(D)



(E)



3. Find the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\cdot\frac{1}{n} =$.
   (A) 0
   (B) $\frac{1}{2}$
   (C) $\frac{2}{3}$
   (D) 1
   (E) ∞

4. Consider the differential equation $\frac{dy}{dx} = y - 2x + 3$, where $y = f(x)$ is the solution to the equation and $f(2) = 5$. Using Euler's method starting at $x_0 = 2$ with step size $\Delta x = 0.5$, what is the approximation for $f(3)$?
   (A) 7
   (B) 8.5
   (C) 9
   (D) 9.5
   (E) 11

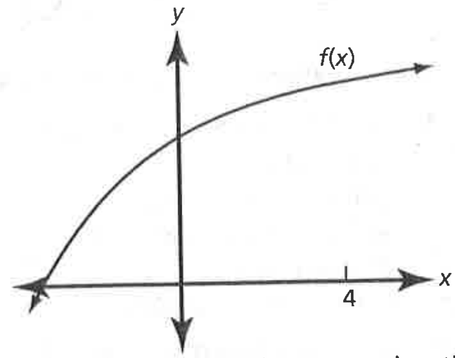
GO ON TO NEXT PAGE

5. The equation of the tangent line to the function $y = 8\sqrt{3x+1}$ at $x = 5$ is
(A) $y = 3x + 27$
(B) $y = x + 27$
(C) $y = 3x + 17$
(D) $y = 6x + 12$
(E) $y = 6x + 2$

6. The position vector for a particle moving in the xy-plane for $t \geq 0$ is $\left(10\ln(1+t),\ 16\sqrt{t}\right)$. The slope of the tangent line to the path of the particle at $t = 4$ is
(A) $\frac{16}{5\ln 5}$
(B) $\frac{1}{2}$
(C) $2\sqrt{5}$
(D) $\frac{8}{5}$
(E) 2

7. Evaluate $\int_0^1 \frac{3}{x}\,dx$.
(A) 0
(B) 1
(C) $3e$
(D) e^3
(E) ∞

8. The graph of $f(x)$ is pictured below. Which of the following statements about $\int_0^4 f(x)\,dx$ are true?



I. A left endpoint approximation is greater than $\int_0^4 f(x)\,dx$.
II. A right endpoint approximation is less than $\int_0^4 f(x)\,dx$.
III. A trapezoidal approximation is less than $\int_0^4 f(x)\,dx$.

(A) None are true
(B) I and II only
(C) III only
(D) I and III only
(E) I, II, and III

9. The general solution to the differential equation $\frac{dy}{dx} = y\left(1 + \frac{1}{x^2}\right)$ is $y =$
(A) $Ce^{\tan^{-1} x}$
(B) $Ce^{x - \frac{1}{x}}$
(C) $e^{x + \frac{1}{x}} + C$
(D) $\sqrt{2x - \frac{2}{x} + C}$
(E) $e^{x - \frac{1}{x}}$

10. What is the slope of the curve $2xy^2 = 3x^2 - y^3$ at the point (1, 1)?
(A) –3
(B) $\frac{1}{7}$
(C) $\frac{4}{7}$
(D) $\frac{6}{7}$
(E) $\frac{6}{5}$

11. If $f'(x) = 12x^2 \sin\left(2x^3 - 16\right)$ and $f(2) = 5$, then $f(x) =$
(A) $-2\cos\left(2x^3 - 16\right) + 7$
(B) $-4x^3\cos\left(2x^3 - 16\right) + 5$
(C) $2\cos\left(2x^3 - 16\right) + 3$
(D) $-2\cos\left(2x^3 - 16\right) + 5$
(E) $24x\cos\left(2x^3 - 16\right) + 5$

12. The first four terms of the Taylor expansion for $f(x)$ about $x = 3$ are $5 - \frac{x-3}{4} - \frac{7(x-3)^2}{3} + \frac{9(x-3)^3}{2}$. What is the value of $f''(3)$?
(A) $-\frac{14}{3}$
(B) $-\frac{7}{3}$
(C) $-\frac{7}{6}$
(D) $-\frac{1}{2}$
(E) $-\frac{1}{4}$

13. The graph of $f(x) = x^6 - 5x^4$ has inflection points at $x =$
(A) $-\sqrt{2}$ and $\sqrt{2}$ only
(B) 0 and $\sqrt{2}$ only
(C) 0 and $\sqrt{\frac{10}{3}}$ only
(D) $-\sqrt{\frac{10}{3}}$, 0, and $\sqrt{\frac{10}{3}}$
(E) $-\sqrt{2}$, 0, and $\sqrt{2}$

14. $\lim_{x \to 0} \frac{\cos x - e^x}{\ln(1+x)} =$
(A) –1
(B) 0
(C) 1
(D) e
(E) ∞

15. Which of the following series converge?

I. $\sum_{k=0}^{\infty} \frac{3^{k+1}}{4^k}$

II. $\sum_{k=0}^{\infty} (-1)^k \frac{k^2}{(2k+1)^2}$

III. $\sum_{k=1}^{\infty} \frac{|\sec k|}{k}$

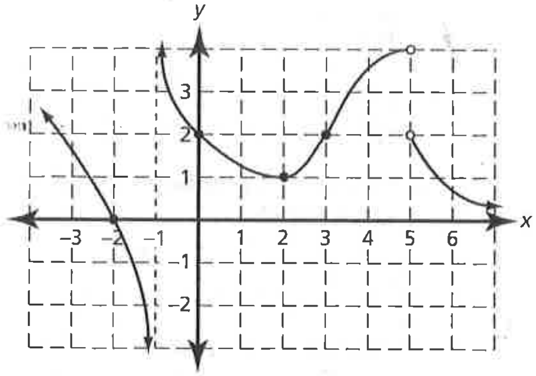
(A) I only
(B) I and II
(C) I and III
(D) II and III
(E) I, II, and III

16. If $\frac{dy}{dt} = k(y-2)$, then $y =$
(A) Ce^{t-2}
(B) $e^{kt} + C$
(C) $\frac{k}{2}(t-2)^2 + C$
(D) $Ce^{kt} + 2$
(E) $\ln|kt + C| + 2$

3

GO ON TO NEXT PAGE

Questions 17 and 18 refer to the following information:



Let $F(x)=\int_0^{2x-1} f(t)\,dt$, where $f(t)$ is pictured above.

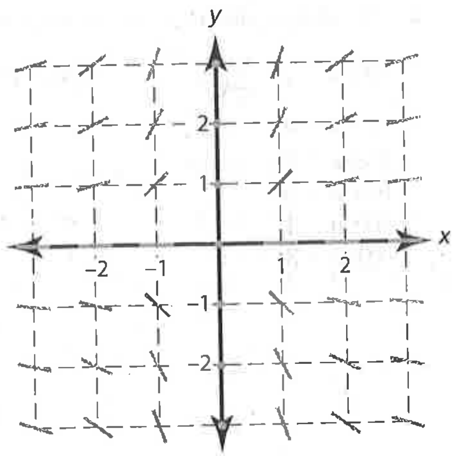
17. What is the domain of $F(x)$?
    (A) All real numbers except -1 and 5
    (B) $-1<x<5$
    (C) $0<x<3$
    (D) $x<-1$
    (E) $x>5$

18. What is the value of $F'(2)$?
    (A) 0
    (B) 1
    (C) 2
    (D) 4
    (E) undefined

19. The slope of the normal line to $f(x)=3\sin^{-1}x$ at $x=0$ is
    (A) $-\frac{1}{3}$
    (B) 0
    (C) $\frac{1}{3}$
    (D) 3
    (E) undefined

20. A particle moves in the xy-plane according to the parametric equations $x=\tan t$ and $y=e^{\frac{1}{2}t}$. An expression for the length of the path of the particle from $t=0$ to $t=1$ is
    (A) $\int_0^1\left[\sec^2 t+\frac{1}{2}e^{\frac{1}{2}t}\right]dt$
    (B) $\int_0^1\sqrt{\tan^2 t+e^t}\,dt$
    (C) $\int_0^1\sqrt{\sec^4 t+\frac{1}{4}e^t}\,dt$
    (D) $\int_0^1\left[\sec^4 t+\frac{1}{4}e^t\right]dt$
    (E) $\int_0^1\sqrt{\sec^2 t+\frac{1}{2}e^{\frac{1}{2}t}}\,dt$

21. Which expression below represents the first four terms of the Maclaurin approximation to the area under the curve $f(x)=e^{x^2}$ from $x=0$ to $x=1$?
    (A) $1+\frac{1}{3}+\frac{1}{10}+\frac{1}{42}$
    (B) $1+1+\frac{1}{4}+\frac{1}{36}$
    (C) $1+1+\frac{1}{2}+\frac{1}{6}$
    (D) $1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}$
    (E) $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$



22. The slope field above represents an approximation to the general solution to which differential equation?

(A) $\frac{dy}{dx} = \frac{y}{x}$

(B) $\frac{dy}{dx} = \frac{x}{y^2}$

(C) $\frac{dy}{dx} = \frac{y}{x^2}$

(D) $\frac{dy}{dx} = \frac{y^3}{x}$

(E) $\frac{dy}{dx} = \frac{y^2}{x^2}$

23. Let $f(x) = x\sin(x)$. The first four nonzero terms of the Taylor approximation for $f'(x)$ about $x = 0$ are

(A) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

(B) $2x - \frac{4x^3}{3!} + \frac{6x^5}{5!} - \frac{8x^7}{7!}$

(C) $x^2 + \frac{x^4}{3!} + \frac{x^6}{5!} + \frac{x^8}{7!}$

(D) $1 + 2x + \frac{3x^2}{2!} + \frac{4x^3}{3!}$

(E) $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!}$

24. The area enclosed by the polar curve $r\cos\frac{1}{2}\theta = 1$ in the interval $0 \le \theta \le \frac{\pi}{2}$ is

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\pi}{4}$

(D) 1

(E) 2

25. The volume of the solid generated by revolving the region enclosed between the graph of $y = 1 + x^2$ and the lines $y = 1$ and $x = 2$ about the x-axis is given by which integral expression?

(A) $\pi\int_0^2 x^4\,dx$

(B) $\pi\int_0^2 (1+x^2)^2\,dx$

(C) $\pi\int_1^5 (1-\sqrt{y-1})^2\,dy$

(D) $\pi\int_0^2 \left[(1+x^2)^2 - 1^2\right]dx$

(E) $2\pi\int_0^2 x^3\,dx$

26. $\int \frac{2x-3}{x^2+9x+18}\,dx =$

(A) $\ln|(x+9)^3(x+2)| + C$

(B) $\ln\left|\frac{(x+6)^5}{(x+3)^3}\right| + C$

(C) $3\ln|x+9| - \ln|x+2| + C$

(D) $\ln|x^2+9x+18| + C$

(E) $5\ln|x+6| + 3\ln|x+3| + C$

5

GO ON TO NEXT PAGE

27. The acceleration vector of a particle moving in the xy-plane is $(-\pi \sin \pi t,\ 2t+1)$, for $t \geq 0$. If the velocity vector at $t = 0$ is (1, 0), then how fast is the particle moving when $t = 2$?
    (A) 5
    (B) 6
    (C) $\sqrt{37}$
    (D) $\sqrt{40}$
    (E) $\sqrt{\pi^4 + 4}$

28. What are all the values of a for which the series $\sum_{k=1}^{\infty} \frac{k^2}{k^{2a-3}+4}$ converges?
    (A) $a > 2$
    (B) $a \geq 3$
    (C) $a < 3$
    (D) $a > 1$
    (E) $a > 3$

Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. E	2. B	3. C	4. D	5. C
6. E	7. E	8. C	9. B	10. C
11. A	12. A	13. A	14. A	15. A
16. D	17. C	18. D	19. A	20. C
21. A	22. C	23. B	24. D	25. D
26. B	27. C	28. E	29. B	30. D
31. D	32. C	33. C	34. E	35. B
36. A	37. C	38. B	39. D	40. E
41. D	42. E	43. A	44. B	45. D

6

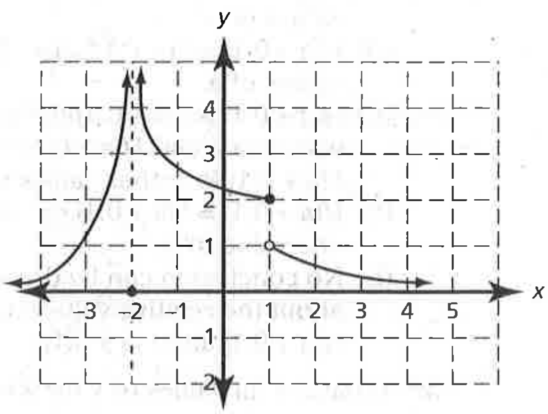
Section I, Part B: Multiple-Choice Questions
Time: 50 minutes
Number of Questions: 17

A graphing calculator may be used on this part of the examination.

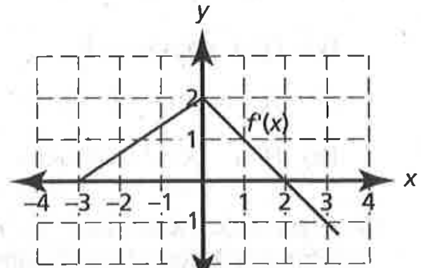
29. Let $f(x)$ be a continuous function defined on the interval $4 \le x \le 10$. A table of selected values of $f(x)$ is given below.

x	$f(x)$
4	24
6	37
8	47
10	58

What is the estimate of $\int_4^{10} f(x)\,dx$ produced by a trapezoidal approximation with $n = 3$?
(A) 216
(B) 250
(C) 262
(D) 270
(E) 284



30. The graph of a function $f(x)$ is shown above. Which of the following statements is true?
(A) $\lim_{x\to -2} f(x)$ exists.
(B) $f(1)$ does not exist.
(C) $f'(1)$ exists.
(D) $\lim_{x\to 1^+} f(x)$ exists.
(E) $\lim_{x\to \infty} f(x)$ does not exist.



31. The graph of $f'(x)$, consisting of a pair of line segments, is pictured above. If $f(-3) = 0$, then $f(3) =$
(A) −1
(B) 3
(C) 4
(D) 4.5
(E) 5.5

32. A particle moves in the xy-plane along the path of the curve $y = x\sin x$ for time $t \ge 0$. When the particle is at the point $(3, 3\sin 3)$, $\frac{dy}{dt} = -2$. What is the value of $\frac{dx}{dt}$ at the same point?
(A) −2.829
(B) 0.423
(C) 0.707
(D) 2.020
(E) 5.658

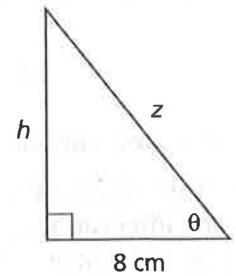
33. Let $f(x)$ be a function defined for $1.6 \le x \le 11.6$ such that $f'(x) = \ln x \sin x$. How many inflection points does the graph of $f(x)$ have on this interval?
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

GO ON TO NEXT PAGE

34. Which of the following functions has the smallest average value on the given interval?
(A) $f(x) = \cos x$ on $0 \le x \le \frac{3\pi}{4}$
(B) $f(x) = \cos 2x$ on $0 \le x \le \pi$
(C) $f(x) = \cos x$ on $0 \le x \le \frac{\pi}{2}$
(D) $f(x) = \sin x$ on $0 \le x \le 2\pi$
(E) $f(x) = \cos 2x$ on $0 \le x \le \frac{3\pi}{4}$

35. A particle moves along a line for time $t \ge 0$ such that its velocity is $v(t) = 10e^{-t}\cos t$. What is the velocity of the particle when its acceleration is zero for the first time?
(A) –2.709
(B) –0.670
(C) 2.356
(D) 3.185
(E) 10.000

36. Which of the following series are conditionally convergent?
I. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^3+1}$
II. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^4+1}$
III. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{k^3+1}$
(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III



37. The base of the right triangle pictured above is 8 centimeters and the angle θ is increasing at the constant rate of 0.03 radians per second. How fast, in centimeters per second, is the altitude h of the triangle increasing when $h = 13$?
(A) 0.458 cm/sec
(B) 0.744 cm/sec
(C) 0.874 cm/sec
(D) 12.626 cm/sec
(E) 29.125 cm/sec

38. Let $f'(x) = e^x + x$ and let $H(x)$ be the equation of the tangent line to $f(x)$ at $x = a$. If $H(x)$ is used to produce an estimate for $f(a + 0.1)$, then which of the following statements is true?
(A) $H(a + 0.1) > f(a + 0.1)$ for all values of a.
(B) $H(a + 0.1) < f(a + 0.1)$ for all values of a.
(C) $H(a + 0.1) > f(a + 0.1)$ for some values of a and $H(a + 0.1) < f(a + 0.1)$ for other values of a.
(D) $H(a + 0.1) = f(a + 0.1)$ for at least one value of a.
(E) No conclusion can be drawn about the relative values of $H(a + 0.1)$ and $f(a + 0.1)$.

39. What are all values of x for which the series $\sum_{k=0}^{\infty}\frac{(2x)^k}{k+1}$ converges?
(A) $x = 0$
(B) $-\frac{1}{2} \le x \le \frac{1}{2}$
(C) $-2 < x < 2$
(D) $-\frac{1}{2} \le x < \frac{1}{2}$
(E) x can be any real number.

8

40. What is the total area enclosed between the graphs of the functions $f(x) = \frac{1}{8}x^3 + \frac{1}{4}x^2 - \frac{5}{2}x + 1$ and $g(x) = \frac{1}{2}x + 1$?
(A) 10.667
(B) 20.833
(C) 31.500
(D) 35.333
(E) 42.167

41. A large auto dealer is running a special sales promotion. They expect to sell cars at the rate of $0.32x^2 - 0.01x^3$ cars per day for the first x days of the sale. According to the model, about how many cars will the dealer sell in the first 30 days of the sale?
(A) 18
(B) 29
(C) 722
(D) 855
(E) 863

x	$f(x)$	$f'(x)$
–1	4	3
–3	–2	7

42. The table above contains values of $f(x)$ and $f'(x)$ for certain values of x. If $g(x) = x^2 f(3x)$, then $g'(-1) =$
(A) –14
(B) 11
(C) 17
(D) 21
(E) 25

43. The base of a certain solid is the region in the first quadrant bounded by the x- and y-axes and the curve $y = 15 - e^x$. If each plane cross section of the solid perpendicular to the x-axis is a semicircle with diameter across the base, then the volume of the solid is
(A) 118.325
(B) 155.287
(C) 236.649
(D) 371.728
(E) 473.299

44. Let $f(x)$ be a continuous function with the properties that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 3$. What is the value of $\lim_{x\to\infty} [f(x)]^{\frac{1}{x}}$?
(A) 0
(B) 1
(C) 3
(D) e^3
(E) ∞

45. Consider the differentiable function $f(x) = \ln x - x + 3$ on the closed interval $0.5 \le x \le 3.5$. What is the value of c in the interval $0.5 < x < 3.5$ that satisfies the Mean Value Theorem?
(A) 1
(B) 1.484
(C) 1.507
(D) 1.542
(E) 2

(9)

Calculus BC—Exam 1
Section II, Part A

Time: 45 minutes
Number of problems: 3

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS IN THIS PART OF THE EXAMINATION.

1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^2 + 4} \qquad \text{and} \qquad \frac{dy}{dt} = 3e^t + 2e^{-t}$$

for all real values of t. At time $t = 0$, the position of the particle is $(3, 4)$.

(A) Find the speed and acceleration vector of the particle at time $t = 0$.

(B) Find the equation of the line tangent to the path of the particle at time $t = 0$.

(C) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 2$.

(D) Find the x-coordinate of the position of the particle at time $t = 2$.

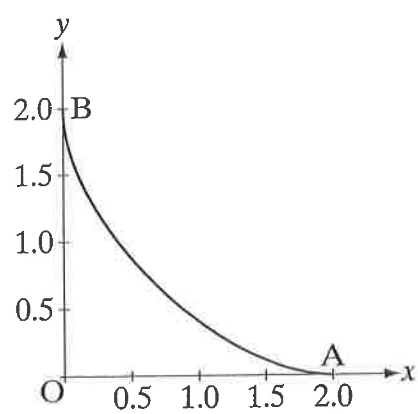
10

6. A particle moves along the curve defined by the equation $y = x^2 - 2x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{t+1}}$, for $t \geq 3$ with the initial condition $x(3) = -1$.

(A) Find $x(t)$ in terms of t.

(B) Find dy/dt in terms of t.

(C) Find the location and speed of the particle at time $t = 8$.



4. Let the coordinate axes represent two highways that meet at right angles. In order to safely connect the two highways, a curved road is to be built from point A to point B as shown in the figure above. The parametric equations of the connecting road are

$x(t) = 2\cos^3 t$ and $y(t) = 2\sin^3 t$ for $0 \leq t \leq \frac{\pi}{2}$, where $x(t)$ and $y(t)$ are in miles.

In order to make the intersections safe, the slope of the curved road should be the same as the slope of each highway where they meet.

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Show that the slope of the curved road and the slope of each highway are the same at points A and B.

(c) Set up but do not evaluate an integral expression which gives the length of the curved road from A to B.

11

2. Two particles move in the xy-plane. For time $0 \leq t \leq 2\pi$, the position of particle A is given by $x(t) = \cos t$ and $y(t) = t$, and the position of particle B is given by $x(t) = \sin t$ and $y(t) = t$.

   (a) In the viewing window provided below, sketch the path of particles A and B. Label the paths A and B and indicate with arrows the direction of each particle along its path.

   (b) Find the velocity vector for each particle.

   (c) At $t = 5$, which particle is moving faster to the right? Justify your answer.

   (d) At $t = 5$, which particle's upward speed is greater? Justify your answer.

---

5. A particle moves along the curve defined by the parametric equations $x(t) = 2t$ and $y(t) = 36 - t^2$ for time t, $0 \leq t \leq 6$. A laser light on the particle points in the direction of motion and shines on the x-axis.

   (a) What is the velocity vector of the particle?

   (b) In terms of t, write an equation of the line tangent to the graph of the curve at the poir $(2t, 36 - t^2)$.

   (c) Express the x-coordinate of the point on the x-axis that the laser light hits as a function of t.

   (d) At what speed is the laser light moving along the x-axis at time $t = 3$? Justify your answer.

Section II Part A: A graphing calculator is required for these problems.

1. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with $x(t) = 2 + \sqrt{t}$ and $\frac{dy}{dt} = te^t - e^t$ for $t \geq 0$.

   (a) Find the speed of the object at time $t = 3$.

   (b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 3$.

   (c) At what time t is the object at the point on the curve where the line tangent to the curve has slope 5?

   (d) At time $t = 0$, the object is at position $(2, -2)$. Find $y(3)$.

AP Calculus BC

CHAPTER 11 WORKSHEET

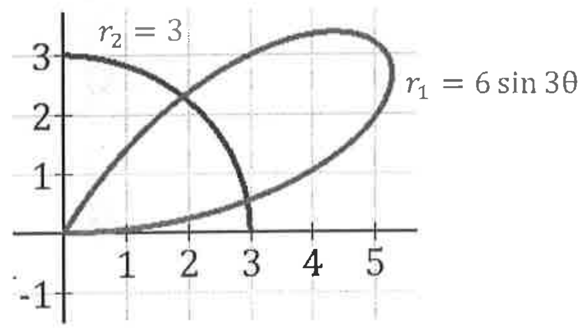
PARAMETRIC EQUATIONS AND POLAR COORDINATES

Name______________________

Seat # ________ Date ______________

Derivatives and Equations in Polar Coordinates

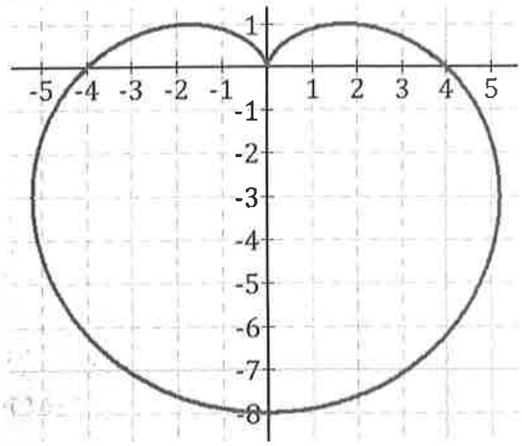
1. The graphs of the polar curves $r_1 = 6\sin 3\theta$ and $r_2 = 3$ are shown to the right.
   (You may use your calculator for all sections of this problem.)
   a) Find the coordinates of the points of intersection of both curves for $0 \le \theta < \frac{\pi}{2}$. Write your answers using polar coordinates.
   b) Write the coordinates of the points of intersection using now rectangular coordinates.



   c) Find $\left.\frac{dr_1}{d\theta}\right|_{\theta=\frac{\pi}{4}}$. Interpret the meaning of your answer in the context of the problem.
   d) For $0 \le \theta < \frac{\pi}{2}$, there are two points on r_1 with x-coordinate equal to 4. Find the subject points. Express your answer using polar coordinates.
   e) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r_1.
   f) Write in terms of x and y an equation for the line tangent to the graph of the curve r_1 at the point where $\theta = \frac{\pi}{4}$.

---

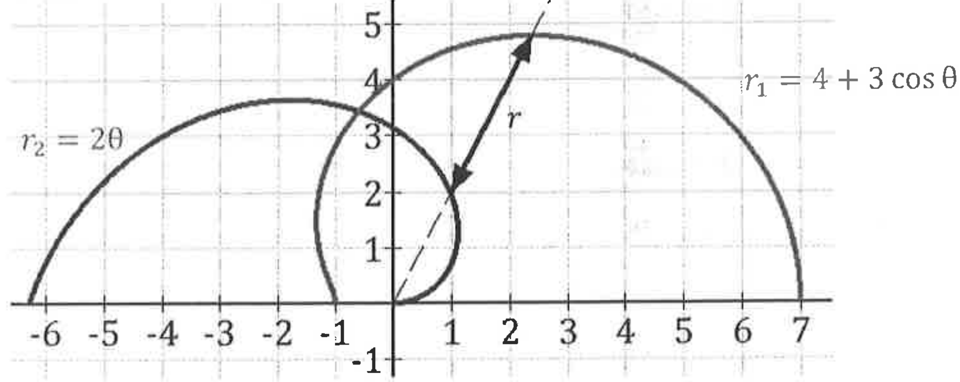
2. The graph of the polar curve $r = 4 - 4\sin\theta$ is shown to the right.
   (You may use your calculator for all sections of this problem.)
   a) For $0 \le \theta < 2\pi$, there are two points on r with y-coordinate equal to -4. Find the subject points. Express your answers using polar coordinates.
   b) Write an expression for the x-coordinate of each point on the graph of $r = 4 - 4\sin\theta$. Express your answer in terms of θ.
   c) A particle moves along the polar curve $r = 4 - 4\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the time interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.



   d) Find $\left.\frac{dr}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.
   e) Find $\left.\frac{dx}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.

---

3. The graphs of the polar curves $r_1 = 4 + 3\cos\theta$ and $r_2 = 2\theta$ are shown below.



(Do NOT use your calculator for this problem unless indicated!)

a) Find the coordinates of the point of intersection of both curves for $0 \le \theta < \pi$. Write your answer using polar coordinates. *(You may use your calculator for this section.)*

b) As the curves are traced, the distance between them, $r(\theta)$, changes (see drawing.) Find an expression for $r(\theta)$ the distance between both curves in the interval $0 \le \theta \le \frac{\pi}{2}$.

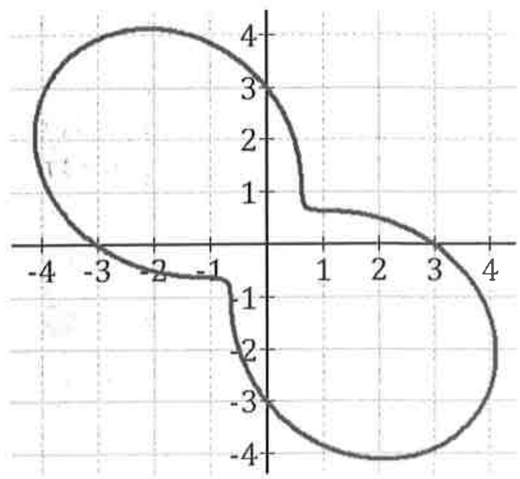
c) Write in terms of θ an expression for $\frac{dr}{d\theta}$. Use your answer to find $\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{3}}$. Interpret the meaning of your answer in the context of the problem.

d) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r_2.

e) Find the coordinates of the point where curve r_2 has a horizontal tangent line in the interval $0 < \theta < \pi$. Write your answer using rectangular coordinates. *(You may use your calculator for this section.)*

---

4. The graph of the polar curve $r = 3 - 2\sin(2\theta)$ for $0 \le \theta < 2\pi$ is shown to the right.
(You may use your calculator for all sections of this problem.)



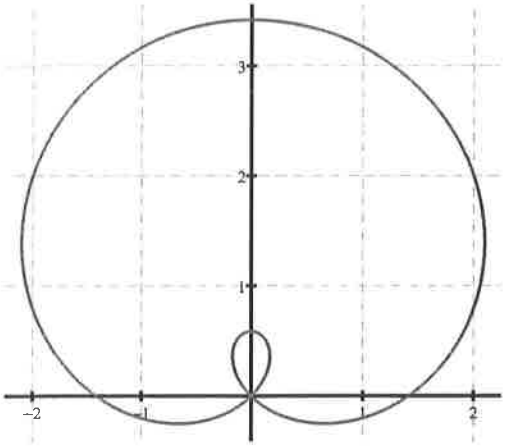
a) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r.

b) Find the coordinates of the point where curve r has a vertical tangent line in the interval $0 \le \theta < \pi$. Write your answer using polar coordinates.

c) Write in terms of x and y an equation for the line tangent to the graph of the curve r at the point where $\theta = \frac{\pi}{6}$.

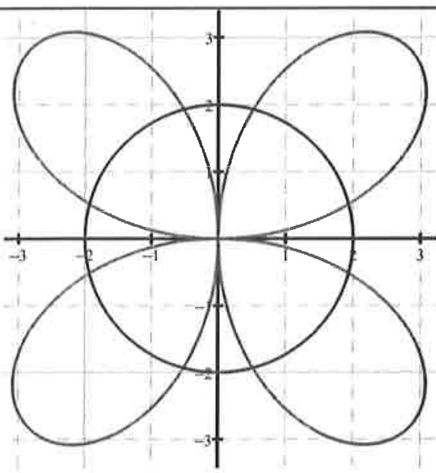
d) A particle moves along the polar curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 2$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$. Interpret the meaning of your answer in the context of the problem.

e) Assume now that for the particle whose motion was described in section (d) we have $\theta = 2t$. Find the position vector of the particle $\langle x(t), y(t)\rangle$ in terms of t. Use your calculator to find the velocity vector and the speed of the particle at $t = 1.5$.



3. The figure to the left shows the graph of $r = \sqrt{2} + 2\sin\theta$ for $0 \le \theta \le 2\pi$.

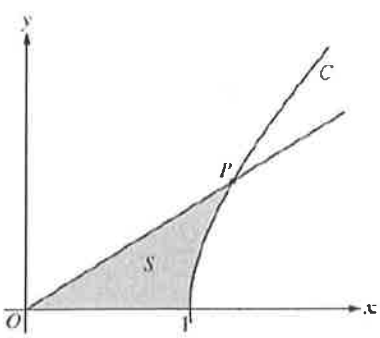
   a) For $0 < \theta < \frac{\pi}{2}$, $\frac{dr}{d\theta}$ is positive. What does this fact say about r? What does this fact say about the curve?
   b) For $0 \le \theta \le 2\pi$, find the angle(s) θ that corresponds to the point on the curve with y-coordinate 3.
   c) Set up an integral to find the area inside the inner loop for the curve $r = \sqrt{2} + 2\sin\theta$. Use your calculator to evaluate your integral.
   d) Set up an integral to find the arc length of the inner loop. Use your calculator to evaluate your integral.
   e) Set up an integral to find the area inside the outer loop for the curve $r = \sqrt{2} + 2\sin\theta$. Use your calculator to evaluate your integral.
   f) Set up an integral to find the arc length of the outer loop. Use your calculator to evaluate your integral.
   g) Find the area of the graph inside the outer loop but outside the inner loop.



4. The figure to the left shows the graphs of $r = 4\sin 2\theta$ and $r = 2$ for $0 \le \theta \le 2\pi$.

   a) Set up two or more integrals to find the area common to both curves in the first quadrant (the area inside both graphs.) Use your calculator to evaluate the integrals and find such area.
   b) Find the total area common to both curves for $0 \le \theta \le 2\pi$.
   c) Set up two or more integrals to find the perimeter of the region common to both curves in the first quadrant. Use your calculator to evaluate the integrals and find the perimeter.

2003 CALCULUS BC



5. The figure to the left shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1+y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

   a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.
   b) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \dfrac{1}{\cos^2\theta - \sin^2\theta}$.
   c) Use the polar equation given in part (b) to set up and integral expression with respect to the polar angle θ that represents the area of S.