AB Practice Examination 1

SECTION I

Part A TIME: 55 MINUTES

The use of calculators is not permitted for this part of the examination. There are 28 questions in Part A, for which 55 minutes are allowed. To compensate for possible guessing, the grade on this part is determined by subtracting one-fourth of the number of wrong answers from the number answered correctly.

Directions: Choose the best answer for each question.

1. $\lim\limits_{x\to\infty}\dfrac{20x^2-13x+5}{5-4x^3}$ is

(A) -5 (B) ∞ (C) 0 (D) 5 (E) 1

2. $\lim\limits_{h\to 0}\dfrac{\ln(2+h)-\ln 2}{h}$ is

(A) 0 (B) $\ln 2$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{\ln 2}$ (E) ∞

3. If $y=e^{-x^2}$, then $y''(0)$ equals

(A) 2 (B) -2 (C) $\dfrac{2}{e}$ (D) 0 (E) -4

Questions 4 and 5. Use the following table, which shows the values of the differentiable functions f and g.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

4. The average rate of change of function f on [1,4] is

(A) 7/6 (B) 4/3 (C) 15/8 (D) 9/4 (E) 8/3

5. If $h(x) = g(f(x))$ then $h'(3) =$

(A) 1/2 (B) 1 (C) 4 (D) 6 (E) 9

6. The derivative of a function f is given for all x by

$$f'(x) = x^2(x+1)^3(x-4)^2.$$

The set of x values for which f is a relative maximum is

(A) $\{0, -1, 4\}$ (B) $\{-1\}$ (C) $\{0, 4\}$
(D) $\{1\}$ (E) none of these

7. If $y = \dfrac{x-3}{2-5x}$, then $\dfrac{dy}{dx}$ equals

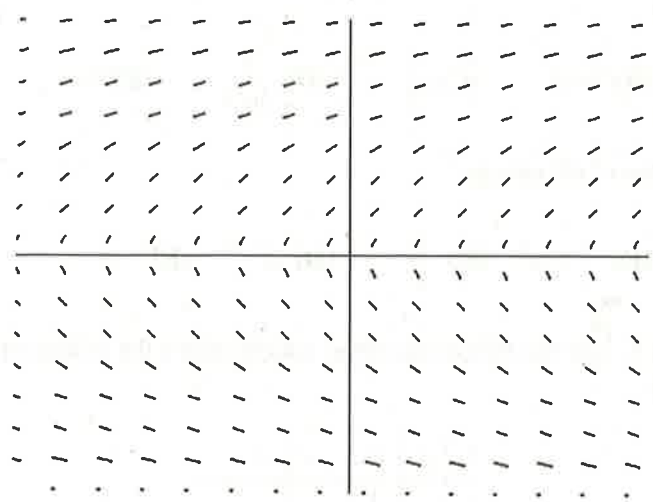
(A) $\dfrac{17-10x}{(2-5x)^2}$ (B) $\dfrac{13}{(2-5x)^2}$ (C) $\dfrac{x-3}{(2-5x)^2}$

(D) $\dfrac{17}{(2-5x)^2}$ (E) $\dfrac{-13}{(2-5x)^2}$

8. The maximum value of the function $f(x) = xe^{-x}$ is

(A) $\dfrac{1}{e}$ (B) e (C) 1 (D) -1 (E) none of these

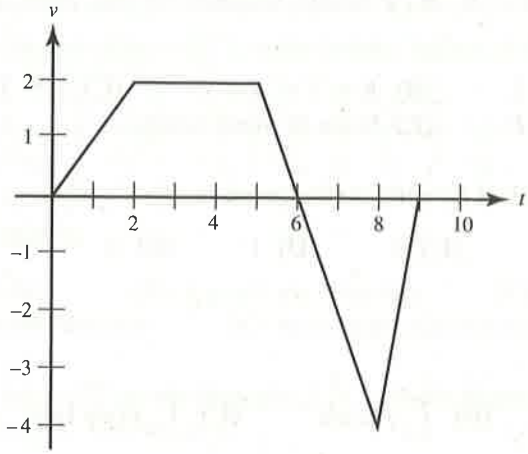
9. Which equation has the slope field shown below?



(A) $\dfrac{dy}{dx} = \dfrac{5}{y}$ (B) $\dfrac{dy}{dx} = \dfrac{5}{x}$ (C) $\dfrac{dy}{dx} = \dfrac{x}{y}$

(D) $\dfrac{dy}{dx} = 5y$ (E) $\dfrac{dy}{dx} = x + y$

Questions 10–12. The graph below shows the velocity of an object moving along a line, for $0 \leq t \leq 9$.



10. At what time does the object attain its maximum acceleration?

(A) $2 < t < 5$ **(B)** $5 < t < 8$ **(C)** $t=6$ **(D)** $t=8$ **(E)** $8 < t < 9$

11. The object is farthest from the starting point at $t=$

(A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

12. At $t=8$, the object was at position $x=10$. At $t=5$, the object's position was $x=$

(A) −5 **(B)** 5 **(C)** 7 **(D)** 13 **(E)** 15

13. $\int_{\pi/4}^{\pi/2} \sin^3 \alpha \cos \alpha \, d\alpha$ is equal to

(A) $\frac{3}{16}$ **(B)** $\frac{1}{8}$ **(C)** $-\frac{1}{8}$ **(D)** $-\frac{3}{16}$ **(E)** $\frac{3}{4}$

14. $\int_0^1 \frac{e^x}{(3-e^x)^2}\,dx$ equals

(A) $3 \ln (e-3)$ **(B)** 1 **(C)** $\frac{1}{3-e}$ **(D)** $\frac{e-2}{3-e}$ **(E)** none of these

15. A differentiable function has the values shown in this table:

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	1.39	1.73	2.10	2.48	2.88	3.30

Estimate $f'(2.1)$.

(A) 0.34 **(B)** 0.59 **(C)** 1.56 **(D)** 1.70 **(E)** 1.91

16. If $A = \int_0^1 e^{-x}dx$ is approximated using Riemann sums and the same number of subdivisions, and if L, R, and T denote, respectively left, right, and trapezoid sums, then it follows that

(A) $R \leqslant A \leqslant T \leqslant L$ (B) $R \leqslant T \leqslant A \leqslant L$ (C) $L \leqslant T \leqslant A \leqslant R$
(D) $L \leqslant A \leqslant T \leqslant R$ (E) None of these is true.

17. The number of vertical tangents to the graph of $y^2 = x - x^3$ is

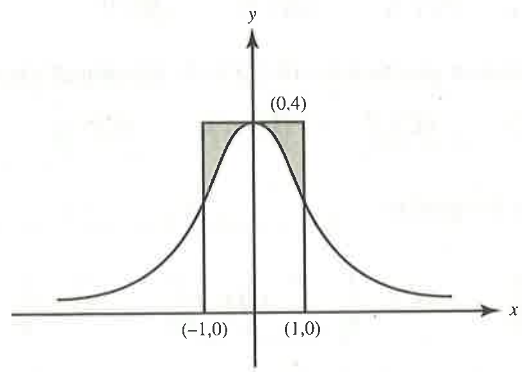
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

18. $\int_0^6 f(x-1)dx =$

(A) $\int_{-1}^7 f(x)dx$ (B) $\int_{-1}^5 f(x)dx$ (C) $\int_{-1}^5 f(x+1)dx$

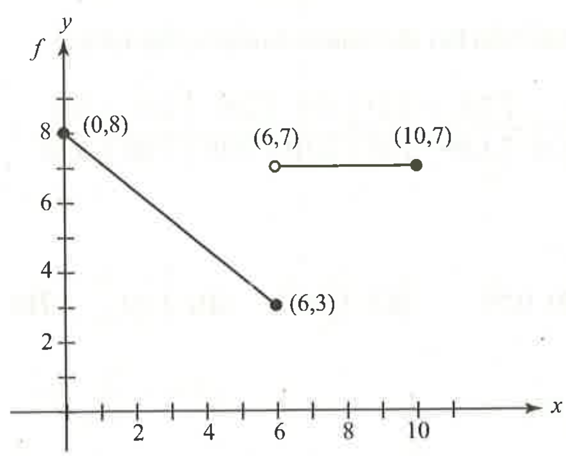
(D) $\int_1^5 f(x)dx$ (E) $\int_1^7 f(x)dx$

19. The equation of the curve shown below is $y = \dfrac{4}{1+x^2}$. What does the area of the shaded region equal?



(A) $4 - \dfrac{\pi}{4}$ (B) $8 - 2\pi$ (C) $8 - \pi$ (D) $8 - \dfrac{\pi}{2}$ (E) $2\pi - 4$

20. Over the interval $0 \le x \le 10$, the average value of the function f shown below



(A) is 6.00. (B) is 6.10. (C) is 6.25.
(D) does not exist, because f is not continuous.
(E) does not exist, because f is not integrable.

21. If $f'(x) = 2f(x)$ and $f(2) = 1$, then $f(x) =$

(A) e^{2x-4} **(B)** $e^{2x}+1-e^4$ **(C)** e^{4-2x} **(D)** e^{2x+1} **(E)** e^{x-2}

22. The table below shows values of $f''(x)$ for various values of x:

x	−1	0	1	2	3
$f''(x)$	−4	−1	2	5	8

The function f could be

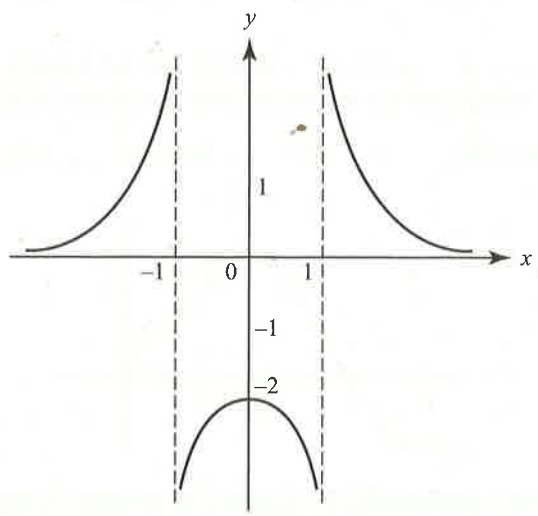
(A) a linear function **(B)** a quadratic function **(C)** a cubic function
(D) a fourth-degree function **(E)** an exponential function

23. The curve $x^3+x\tan y=27$ passes through (3,0). Use the tangent line there to estimate the value of y at $x=3.1$. The value is

(A) −2.7 **(B)** −0.9 **(C)** 0 **(D)** 0.1 **(E)** 3.0

24. At what value of h is the rate of increase of $\sqrt{h}$ twice the rate of increase of h?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{4}$ **(C)** 1 **(D)** 2 **(E)** 4



25. The graph of a function $y = f(x)$ is shown above. Which is true?

(A) $\lim_{x\to 1} f(x) = -\infty$ **(B)** $\lim_{x\to -\infty} f(x) = \pm 1$ **(C)** $\lim_{x\to -2} f(x) = 0$

(D) $\lim_{x\to \infty} f(x) = 0$ **(E)** $\lim_{x\to 0} f(x) = \infty$

26. A function $f(x)$ equals $\frac{x^2 - x}{x - 1}$ for all x except $x = 1$. For the function to be continuous at $x=1$, the value of $f(1)$ must be

(A) 0 **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

27. The number of inflection points on the graph of $f(x) = 3x^5 - 10x^3$ is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

28. Suppose $f(x) = \int_0^x \frac{4+t}{t^2+4}\,dt$. It follows that

(A) f increases for all x
(B) f increases only if $x < -4$
(C) f has a local min at $x = -4$
(D) f has a local max at $x = -4$
(E) f has no critical points

STOP

END OF PART A, SECTION I

Part B TIME: 50 MINUTES

Some questions in this part of the examination require the use of a graphing calculator. There are 17 questions in Part B, for which 50 minutes are allowed. The deduction for incorrect answers on this part is the same as that for Part A.

Directions: Choose the best answer for each question. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

29. Let $G(x) = [f(x)]^2$. At $x=a$, the graph of f is increasing and concave downward, while G is decreasing. Which describes the graph of G at $x=a$?

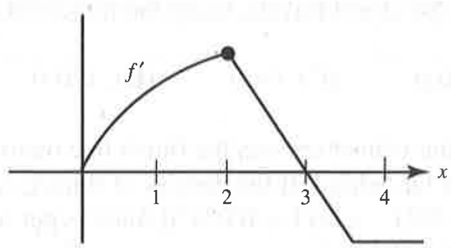
(A) concave downward **(B)** concave upward **(C)** linear
(D) point of inflection **(E)** none of these

30. The value of c for which $f(x) = x + \dfrac{c}{x}$ has a local minimum at $x=3$ is

(A) -9 **(B)** -6 **(C)** -3 **(D)** 6 **(E)** 9

31. An object moving along a line has velocity $v(t)=t\cos t - \ln(t+2)$, where $0 \leq t \leq 10$. The object achieves its maximum speed when t is approximately

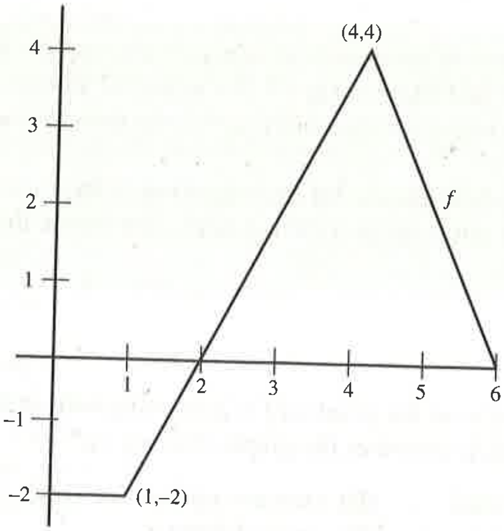
(A) 3.7 **(B)** 5.1 **(C)** 6.4 **(D)** 7.6 **(E)** 9.5



32. The graph of f', which consists of a quarter-circle and two line segments, is shown above. At $x=2$ which of the following statements is true?

(A) f is not continuous.
(B) f is continuous but not differentiable.
(C) f has a relative maximum.
(D) The graph of f has a point of inflection.
(E) none of these

33. Let $H(x) = \int_0^x f(t)\, dt$, where f is the function whose graph appears below.



The local linearization of $H(x)$ near $x=3$ is $H(x) \approx$

(A) $-2x + 8$ **(B)** $2x - 4$ **(C)** $-2x + 4$ **(D)** $2x - 8$ **(E)** $2x - 2$

34. The table shows the speed of an object, in feet per second, at various times during a 6-second interval.

time (sec)	0	1	4	6
speed (ft/sec)	30	22	12	0

Estimate the distance the object travels, using the trapezoid method.

(A) 89 ft **(B)** 90 ft **(C)** 96 ft **(D)** 120 ft **(E)** 147 ft

35. In a marathon, when the winner crosses the finish line many runners are still on the course, some quite far behind. If the density of runners x miles from the finish line is given by $R(x)=20[1 - \cos(1 + 0.03x^2)]$ runners per mile, how many are within 8 miles of the finish line?

(A) 30 **(B)** 145 **(C)** 157 **(D)** 166 **(E)** 195

36. Which best describes the behavior of the function $y = \text{arc}\tan\left(\frac{1}{\ln x}\right)$ at $x=1$?

(A) It has a jump discontinuity.
(B) It has an infinite discontinuity.
(C) It has a removable discontinuity.
(D) It is both continuous and differentiable.
(E) It is continuous but not differentiable.

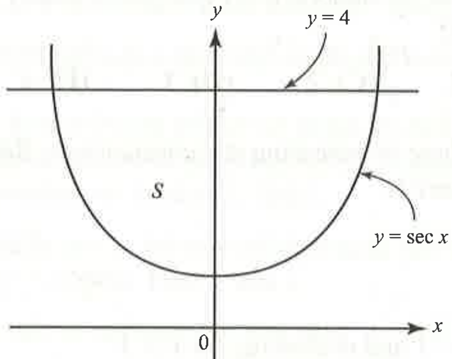
37. If $f(t) = \int_0^{t^2} \frac{1}{1+x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1+t^2}$ **(B)** $\frac{2t}{1+t^2}$ **(C)** $\frac{1}{1+t^4}$ **(D)** $\frac{2t}{1+t^4}$ **(E)** $\tan^{-1} t^2$

38. $\int (\sqrt{x}-2)x^2 dx =$

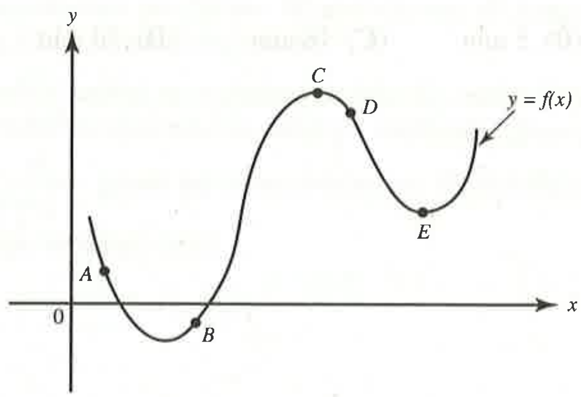
(A) $\frac{2}{3}x^{3/2} - 2x + C$ **(B)** $\frac{5}{2}x^{3/2} - 4x + C$ **(C)** $\frac{2}{3}x^{3/2} - 2x + \frac{x^3}{3} + C$

(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + C$ **(E)** $\frac{2}{7}x^{7/2} - \frac{2}{3}x^3 + C$



39. The region S in the figure shown above is bounded by $y = \sec x$ and $y = 4$. What is the volume of the solid formed when S is rotated about the x-axis?

(A) 0.304 **(B)** 39.867 **(C)** 53.126 **(D)** 54.088 **(E)** 108.177



40. At which point on the graph of $y = f(x)$ shown above is $f'(x) < 0$ and $f''(x) > 0$?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

41. Let $f(x) = x^5 + 1$, and let g be the inverse function of f. What is the value of $g'(0)$?

(A) -1 **(B)** $\frac{1}{5}$ **(C)** 1 **(D)** $g'(0)$ does not exist.

(E) $g'(0)$ cannot be determined from the given information.

42. The hypotenuse AB of a right triangle ABC is 5 feet, and one leg, AC, is decreasing at the rate of 2 feet per second. The rate, in square feet per second, at which the area is changing when $AC=3$ is

(A) $\frac{25}{4}$ **(B)** $\frac{7}{4}$ **(C)** $-\frac{3}{2}$ **(D)** $-\frac{7}{4}$ **(E)** $-\frac{7}{2}$

43. At how many points on the interval $[0,\pi]$ does $f(x) = 2\sin x + \sin 4x$ satisfy the Mean Value Theorem?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

44. If the radius r of a sphere is increasing at a constant rate, then the rate of increase of the volume of the sphere is

(A) constant
(B) increasing
(C) decreasing
(D) increasing for $r < 1$ and decreasing for $r > 1$
(E) decreasing for $r < 1$ and increasing for $r > 1$

45. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?

(A) 2 min **(B)** 5 min **(C)** 18 min **(D)** 20 min **(E)** 40 min

STOP

END OF SECTION I