Key

Test Review with answers:

1.

A An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x-2y=0$ (B) $x-y=0$ (C) $x=0$ (D) $y=0$ (E) $\pi x-2y=0$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$$

$$y'(0) = \frac{1}{2}$$

$$y(0) = 0$$

$$y-0 = \frac{1}{2}(x-0)$$

$$y = \frac{1}{2}x$$

$$2y = x$$

$$x-2y=0$$

2.

B The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

$$y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x} = \frac{2}{e^2}$$

3.

$\frac{d}{dx}(\arcsin 2x) =$

D (A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

$$\frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

4.

$\int \frac{5}{1+x^2}\,dx =$

D (A) $\frac{-10x}{(1+x^2)^2}+C$ (B) $\frac{5}{2x}\ln(1+x^2)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln(1+x^2)+C$

$$5 \cdot \int \frac{1}{1+x^2}\,dx = 5 \cdot \arctan x + C$$

5.

If $\tan(xy) = x$, then $\frac{dy}{dx} =$

E (A) $\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy)-y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy)-y}{x}$

$$\sec^2(xy) \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 1$$

$$x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = 1$$

$$\frac{1-y\sec^2(xy)}{x\sec^2(xy)} = \frac{dy}{dx}$$

$$\left(\frac{1-y\cdot\frac{1}{\cos^2(xy)}}{x\cdot\frac{1}{\cos^2(xy)}}\right)\frac{\cos^2(xy)}{\cos^2(xy)} = \frac{\cos^2(xy)-y}{x}$$

6.

If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

A (A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2\sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

$$\frac{1}{1+\cos^2 x} \cdot (-\sin x)$$

7.

Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

A (A) $\arcsin\frac{x}{5}+C$ (B) $\arcsin x+C$ (C) $\frac{1}{5}\arcsin\frac{x}{5}+C$

(D) $\sqrt{25-x^2}+C$ (E) $2\sqrt{25-x^2}+C$

8.

$\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

D (A) $\frac{1}{9}(3x^2+5)^{\frac{3}{2}}+C$ (B) $\frac{1}{4}(3x^2+5)^{\frac{3}{2}}+C$ (C) $\frac{1}{12}(3x^2+5)^{\frac{1}{2}}+C$

(D) $\frac{1}{3}(3x^2+5)^{\frac{1}{2}}+C$ (E) $\frac{3}{2}(3x^2+5)^{\frac{1}{2}}+C$

$$u = 3x^2+5$$

$$du = 6x\,dx$$

$$\frac{1}{6}\int u^{-1/2}\,du$$

$$\frac{1}{6} \cdot 2 \cdot u^{1/2} + C$$

$$\frac{1}{3}\sqrt{3x^2+5} + C$$

9.

C $\frac{d}{dx}(2^x) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

$$\ln 2(2^x)$$

10.

E If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

$$e^{3\ln(x^2)} = \cancel{e^{\ln}}(x^2)^3$$

$$= x^6$$

$$\frac{d}{dx}(x^6) = 6x^5$$

11.

A $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

$$\arcsin\frac{x}{2}\Big|_0^{\sqrt{3}}$$

$$\arcsin\frac{\sqrt{3}}{2} - \arcsin 0$$

$$\frac{\pi}{3}$$

12.

B If $y = \arctan(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\frac{2e^{2x}}{1+e^{4x}}$ (C) $\frac{e^{2x}}{1+e^{4x}}$ (D) $\frac{1}{\sqrt{1-e^{4x}}}$ (E) $\frac{1}{1+e^{4x}}$

$$\frac{1}{1+(e^{2x})^2} \cdot 2e^{2x}$$

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$1.6(1.5)$ $1.1(3.1)$
$1.5(2.2)$ $1.2(4.3)$

13. B

The table shows the rate in liters/minute at which water leaked out of a container.

Time (min)	0	1.2	2.3	3.8	5.4
Rate (liters/min)	5.6	4.3	3.1	2.2	1.5

A right hand Riemann sum is computed using the four subintervals indicated by the data in the table. This Riemann sum estimates the total amount of water that has leaked out of the container. What is the estimate?

A) 12.70 l B) 14.27 l C) 16.7 l D) 16.95 l E) 19.62 l

14.

B $\int x(x^2-1)^4\,dx =$

$u = x^2 - 1$ $\frac{1}{2}du = x\,dx$

$du = 2x\,dx$

a) $\frac{1}{10}(x^2)(x^2-1)^5$ b) $\frac{1}{10}(x^2-1)^5 + C$ c) $\frac{1}{5}(x^3-x)^5 + C$

d) $\frac{1}{5}(x^2-1)^5 + C$ e) $\frac{1}{2}(x^2-1)^5 + C$

$\frac{1}{2}\int u^4\,du = \frac{1}{2}\cdot\frac{1}{5}u^5 + c$

$\frac{1}{10}(x^2-1)^5 + c$

15.

A $\int \frac{dx}{16+x^2} =$ $\frac{1}{4}\arctan\left(\frac{x}{4}\right) + c$

a) $\frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$ b) $\frac{1}{16}\tan^{-1}\left(\frac{x}{16}\right) + C$ c) $4\tan^{-1}\left(\frac{x}{16}\right) + C$

d) $\frac{1}{4}\sec^2\left(\frac{x}{4}\right) + C$ e) $\frac{1}{4}\tan^{-1}(4x) + C$

16.

Evaluate: $\int \frac{7}{x^2+10x+34}\,dx$

$x^2 + 10x + \boxed{25} + 34 - \boxed{25}$

C

a) $7\ln|x^2+6x+13| + C$ b) $7\left(\frac{x^3}{3} + 5x^2 + 34x\right) + C$

c) $\frac{7}{3}\tan^{-1}\left(\frac{x+5}{3}\right) + C$ d) $-\frac{7}{x} + \frac{7}{5}\ln|x| + \frac{7}{10}x + C$

e) $\frac{17}{x^3} + \frac{7}{3x^2} + \frac{5}{10x} + C$

$(x+5)^2 + 9$

$7\cdot\int \frac{1}{(x+5)^2+9}\,dx$

$7\cdot\frac{1}{3}\arctan\left(\frac{x+5}{3}\right) + C$

17.

$\int \frac{9\,dx}{\sqrt{4-x^2}} =$

a) $\sin^{-1}\frac{x}{2} + C$ b) $4\sin^{-1}\frac{x}{2} + C$ c) $4\tan^{-1}\frac{x}{2} + C$

d) $9\sin^{-1}\frac{x}{2} + C$ e) $9\cos^{-1}\frac{x}{2} + C$

D

$9\cdot\int \frac{1}{\sqrt{4-x^2}}\,dx$ $9\arcsin\left(\frac{x}{2}\right) + c$

18. Let $f(x) = 2e^{3x}$ and $g(x) = 5x^3$. At what value of x do the graphs of f and g have parallel tangents (calculator)?

B

(A) -0.445
(B) -0.366
(C) -0.344
(D) -0.251
(E) -0.165

$f'(x) = 2\cdot e^{3x}\cdot 3$ $g'(x) = 15x^2$

$6e^{3x} = 15x^2$

19. If $f(x) = 2e^x - \frac{5}{3}x^3$, which of the following values of x does f have a relative maximum value (calculator)?

C

$f'(x) = 2e^x - 5x^2 = 0$

a) -0.494
b) 0.259
c) 1.092
d) 2.543
e) 3.310

20. A particle moves in a line with velocity. $v(t) = 3t^2 - e^t$ What is the average velocity of the particle in the closed interval [0, 2]?

B

(A) $\frac{8-e^2}{2}$

(B) $\frac{9-e^2}{2}$

(C) $\frac{11-e^2}{2}$

(D) $\frac{13-e^2}{2}$

(E) $13 - e^2$

$\frac{1}{2}\int 3t^2 - e^t\,dt$

$\frac{1}{2}\cdot(t^3 - e^t)\Big|_0^2$

$\frac{1}{2}\left[8 - e^2 - (0 - e^0)\right]$

$\frac{1}{2}(8 - e^2 + 1)$

1. A	2. B	3. D	4. D	5. E
6. A	7. A	8. D	9. C	10. E
11. A	12. B	13. B	14. B	15. A
16. C	17. D	18. B	19. C	20. B

$\frac{9-e^2}{2}$