Key

MA241 Practice on Series – Support your answer

1. Geometric Series – Converge or Diverge? If Converges, find the sum.

a.) $3+\frac{3}{4}+\ldots+\frac{3}{4^{n-1}}+\ldots$ C

b.) $\sum_{n=1}^{\infty} 2^{-n}3^{n-1}$ D

2. Nth-Term Test - Test for Divergence – If inconclusive, use another test.

a.) $\sum_{n=1}^{\infty}\frac{3n}{5n-1}$ D

b.) $\sum_{n=1}^{\infty}\frac{1}{n^2+3}$ C

3. Telescoping – Converge or Diverge? If Converges, find the sum.

a.) $\sum_{n=1}^{\infty}\frac{5}{(n+2)(n+3)}$ C $\frac{5}{3}$

4. P-Series – Convergent or Divergent?

a.) $\sum_{n=1}^{\infty}\frac{1}{n^6}$ C

b.) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^2}}$ D

5. Integral Test – Converge or Diverge?

a.) $\sum_{n=1}^{\infty}\frac{1}{(3+2n)^2}$ C

b.) $\sum_{n=1}^{\infty}\frac{1}{4n+7}$ D

6. Comparison Test – Converge or Diverge?

a.) $\sum_{n=1}^{\infty}\frac{1}{n^4+n^2+1}$ C

b.) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$ C

7. Limit Comparison – Converge or Diverge?

a.) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+4}$ D

b.) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{4n^3-5n}}$ C

8. Alternating Series – Absolute Convergent, Convergent, or Divergent?

a.) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ conditionally convergent

b.) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^2+1}$ D

9. Ratio Test – Absolute Convergent, Convergent, Divergent?

a.) $\sum_{n=1}^{\infty}\frac{n!}{3^n}$ D

b.) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$ D

10. Root Test

MA241 Practice on Series – Support your answer

Determine which series converge and which diverge. Name the Test you use and support your answer!

1. $\sum_{n=1}^{\infty} \frac{1}{10^n}$ C

2. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$ C

3. $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ C

4. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ D

5. $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$ C

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}+1}{n+1}$ D

7. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^3+1}$ C

8. $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ D

9. $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$ C

MA241 KEY

1) a) $\frac{3}{4^{n-1}} = \frac{3}{4^n \cdot 4^{-1}} = 12 \cdot \left(\frac{1}{4}\right)^n$ $r = \frac{1}{4}$ geo conv

Sum $= \frac{a}{1-r} = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = 4$

b) $2^{-n} \cdot 3^{n-1} = \frac{1}{2^n} \cdot 3^n \cdot 3^{-1} = \frac{1}{3}\left(\frac{3}{2}\right)^n$ $r = \frac{3}{2}$ geo div

2) a) $\lim_{n\to\infty} \frac{3n}{5n-1} = \frac{3}{5}$ $\therefore$ div

b) $\lim_{n\to\infty} \frac{1}{n^2+3} = 0$ $\therefore$ inconclusive

Compare to $\frac{1}{n^2}$ big conv w/ p-series $\therefore$ DCT $\therefore$ conv

3) a) $\frac{5}{(n+2)(n+3)} = \frac{A}{n+2} + \frac{B}{n+3} \rightarrow \frac{5}{n+2} - \frac{5}{n+3}$

$5 = A(n+3) + B(n+2)$

$n=-3$: $5 = B(-1)$ $B = -5$

$n=-2$: $5 = 1A$ $A = 5$

$\left(\frac{5}{3} - \cancel{\frac{5}{4}}\right) + \left(\cancel{\frac{5}{4}} - \cancel{\frac{5}{5}}\right) + \left(\cancel{\frac{5}{5}} - \frac{5}{6}\right) + \ldots$

Sum $= \frac{5}{3}$

4) a) $\frac{1}{n^6}$ conv $p = 6 > 1$

b) $\frac{1}{n^{2/3}}$ div $p = \frac{2}{3} < 1$

5) a) $\int_1^\infty \frac{1}{(3+2x)^2}dx$ $u=3+2x$ $du=2dx$ $\frac{1}{2}du=dx$ $\frac{1}{2}\int u^{-2}du = -\frac{1}{2u}$

$\lim_{b\to\infty} \frac{-1}{2(3+2x)}\Big|_1^b = \frac{-1}{2(3+2b)} - \left(\frac{-1}{2(3+2)}\right)$ $\therefore$ conv

b) $\int_1^\infty \frac{1}{4x+7}dx$ $u=4x+7$ $du=4dx$ $\frac{1}{4}du=dx$ $\frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|u|$

$\lim_{b\to\infty} \frac{1}{4}\ln|4x+7|\Big|_1^b = \frac{1}{4}\ln|4b+7| - \frac{1}{4}\ln 11 = \infty$ $\therefore$ div

6) a) $\frac{1}{n^4+n^2+1}$ compare to: $\frac{1}{n^4}$ big conv p-series

$\therefore$ conv

b) $\frac{1}{n3^n}$ Compare to: $\frac{1}{3^n} = \left(\frac{1}{3}\right)^n$ big conv geo

$\therefore$ conv

7) a) $\frac{\sqrt{n}}{n+4}$ compare to: $\frac{1}{\sqrt{n}}$ div by pseries

$\lim_{n\to\infty} \frac{\sqrt{n}}{n+4} \cdot \frac{\sqrt{n}}{1} = 1$ $\therefore$ div

b) $\frac{1}{\sqrt{4n^3-5n}}$ compare to: $\frac{1}{2n^{3/2}}$ conv pseries

$\lim_{n\to\infty} \frac{1}{\sqrt{4n^3-5n}} \cdot \frac{2n^{3/2}}{1} = 1$ $\therefore$ conv

4) $\frac{\ln n}{n}$ $\int_1^\infty \frac{\ln x}{x}dx$ $u=\ln x$ $du=\frac{1}{x}dx$ $\int u\,du = \frac{u^2}{2}$

$\lim_{b\to\infty} \frac{(\ln x)^2}{2}\Big|_1^b = \frac{(\ln b)^2}{2} - 0 = \infty$ ∴ div integral

5) $\frac{2^n}{3^n}$ $\left(\frac{2}{3}\right)^n$ conv geo

6) $\frac{(-1)^{n+1}\sqrt{n}}{n+1} + \frac{1}{n+1}$

↳ compare to $\frac{1}{n}$ div p-series

$\lim_{n\to\infty} \frac{1}{n+1}\cdot\frac{n}{1} = 1$ ∴ div by LCT

conv + div = div series

7) $\frac{(-1)^{n+1}n}{n^3+1}$ $\lim_{n\to\infty}\frac{n}{n^3+1} = 0$ ✓ $\frac{n+1}{(n+1)^3+1} \le \frac{n}{n^3+1}$ ✓

$\sum_{n=1}^{\infty}\frac{n}{n^3+1}$ compare to: $\frac{1}{n^2}$ conv p-series

$\lim_{n\to\infty}\frac{n}{n^3+1}\cdot\frac{n^2}{1} = 1$ ∴ absolutely conv AST

8) $\frac{1}{\ln(n+1)}$ compare to: $\frac{1}{n+1}$ small div

↳ compare to: $\frac{1}{n}$

$\lim_{n\to\infty}\frac{1}{n+1}\cdot\frac{n}{1} = 1$

↳ ∴ div by LCT

9) $\frac{2^{n+1}}{5^n} = \frac{2^n\cdot 2}{5^n} = \left(\frac{2}{5}\right)^n\cdot 2$ conv geo

8) a) $\frac{(-1)^{n+1}}{2n-1}$ $\lim_{n\to\infty} \frac{1}{2n-1} = 0$ ✓ $\frac{1}{2(n+1)-1} \le \frac{1}{2n-1}$ ✓

$\sum_{n=1}^{\infty} \frac{1}{2n-1}$ Compare to: $\frac{1}{2n}$ small div p-series

∴ div

→ conditionally conv

b) $\frac{(-1)^n \cdot n^2}{n^2+1}$ $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$ ∴ div

9) a) $\frac{n!}{3^n}$ $\lim_{n\to\infty} \left| \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!} \right|$

$\frac{(n+1)(n!)}{3^n \cdot 3} \cdot \frac{3^n}{n!}$ $\lim_{n\to\infty} \left(\frac{n+1}{3}\right) = \infty$ ∴ div

b) $\frac{2^n}{n^2}$ $\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n} \right|$

$\frac{2^n \cdot 2}{(n+1)^2} \cdot \frac{n^2}{2^n} = 2$ ∴ div

1) $\frac{1}{10^n} = \left(\frac{1}{10}\right)^n$ conv geo

2) $\frac{\sin^2 n}{2^n}$ Compare to: $\frac{1}{2^n} = \left(\frac{1}{2}\right)^n$ big conv geo

∴ conv by DCT

3) $\frac{n^3}{2^n}$ $\lim_{n\to\infty} \left| \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3} \right|$ $\frac{(n+1)^3}{2 \cdot 2^n} \cdot \frac{2^n}{n^3} = \frac{1}{2} < 1$

∴ conv ratio