Logistic Growth Name: ______________________________

AP Calculus AB Date: ____________________

Complete the following on a separate sheet of paper. Use a calculator for problems 4b, 4c, 5c and 10 only.

1. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.
   a. If P(0) = 100, then $\lim_{t\to\infty} P(t)$ = _______________. Sketch the graph of P(t).
   b. If P(0) = 1500, then $\lim_{t\to\infty} P(t)$ = _______________. Sketch the graph of P(t).
   c. If P(0) = 3000, then $\lim_{t\to\infty} P(t)$ = _______________. Sketch the graph of P(t).
   d. How many bears are in the park when the population of bears is growing the fastest?
2. The population P(t) of a species satisfies the logistic differential equation, $\frac{dP}{dt} = P(2 - \frac{P}{5000})$, where the initial population is P(0) = 3000 and t is the time in years. What is the $\lim_{t\to\infty} P(t)$?
   a. 2500 b. 3000 c. 4200 d. 5000 e. 10,000
3. Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt} = 3P - 0.01P^2$ where P is the number of wolves at time t in years. Which of the following statements are true?
   I. $\lim_{t\to\infty} P(t)$ = 300
   II. The growth rate of the wolf population is greatest at P = 150.
   III. If P > 300, the population of wolves is increasing.
   a. I only b. II only c. I and II only d. II and III only e. I, II, and III only
4. A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.
   a. If P(0) = 20, solve for P as a function of t.
   b. Using the answer from part a, find P when t = 3 years
   c. Using the answer from part a, find t when P = 80 animals
5. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9 AM.
   a. How many students have heard the rumor when it is spreading the fastest?
   b. If P(0) = 5, solve for P as a function of t.
   c. Use you answer in part b to determine how many hours have passed when half the student body has heard the rumor.
6. Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks. What is the carrying capacity?
7. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt} = P\left(3 - \frac{P}{2000}\right)$. What is $\lim_{t\to\infty} P(t)$? What does the limit represent in the context of this problem?
8. Suppose you are in charge of stocking a fish pond with fish for which the rate o population growth is modeled by the differential equation $\frac{dP}{dt} = 0.08P - 0.0002P^2$
   a. If P(0) = 50, then $\lim_{t\to\infty} P(t)$ = ___________. Sketch the graph of P(t)

b. If P(0) = 300, then $\lim_{t\to\infty} P(t)$ = ____________. Sketch the graph of P(t)

c. If P(0) = 500, then $\lim_{t\to\infty} P(t)$ = ____________. Sketch the graph of P(t)

d. Which of the graphs above, has an inflection point? Which are increasing? Which are decreasing?

9. A certain national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park presently. The population growth of bears can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.1P - 0.001P^2$, where t is measured in years.

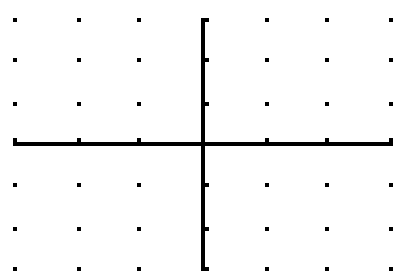
a. Solve for P as a function of t

b. Use the solution found in part a to find the number of bears in the park when t = 3 years.

c. Use the solution found in part a to find how many years it will take for the bear population to reach 50 bears.

10. Consider the initial-value problem $\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$ $\quad y(0) = 2$

a. Draw a slope field to represent the differential.



b. Solve the differential equation

c. Use Euler's method with step size of Δ = 0.1 to obtain an approximation for y(½) (show all work)

d. Find the exact value of y(1/2) (show all work)

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Answers

Answers to Worksheet on Logistic Growth

1. (a) 2500
   (b) 2500
   (c) 2500
   (d) 1250
2. 10,000
3. C
4. (a) $P = \frac{100e^t}{e^t + 4}$ or $P = \frac{100}{1 + 4e^{-t}}$
   (b) 83.393 animals
   (c) 2.773 years
5. (a) 5000 students
   (b) $P = \frac{2000e^{6t}}{e^{6t} + 399}$ or $P = \frac{2000}{1 + 399e^{-6t}}$
   (c) 0.998 hours

6. (a) 100

7. 6000

8a 400

8b 400

8c 400

8d all has an inflection point, a & b are increasing, c is decreasing

9a) $P = \frac{100}{1+be^{-kt}}$

b) 13 years

c) t = 21.972

10 b) y = $2e^{arcsinx}$

10 c) 3.2664

10d) 3.37618