

Day 4 – Limits – Let's Begin the Real Calculus!!!

KEY

Objective: To find a limit numerically, graphically and algebraically.

Informal Definition of a Limit – If $f(x)$ becomes arbitrarily close to a single number, L as x approaches from either side, the limit of $f(x)$, as x approaches c , is L . The limit is written as: $\lim_{x \rightarrow c} f(x) = L$

Finding Limits Numerically

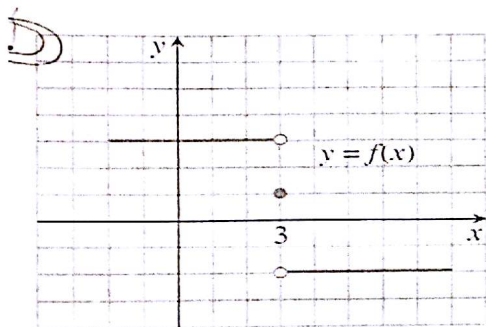
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

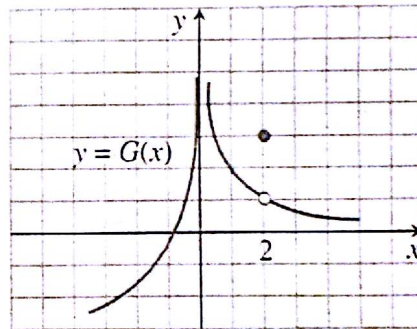
x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	0.99833	0.99998	1	1	0.99998	0.99833

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
f(x)	-0.05	-0.005	-0.0005	-0.0005	0.005	0.04996

Finding Limits Graphically:

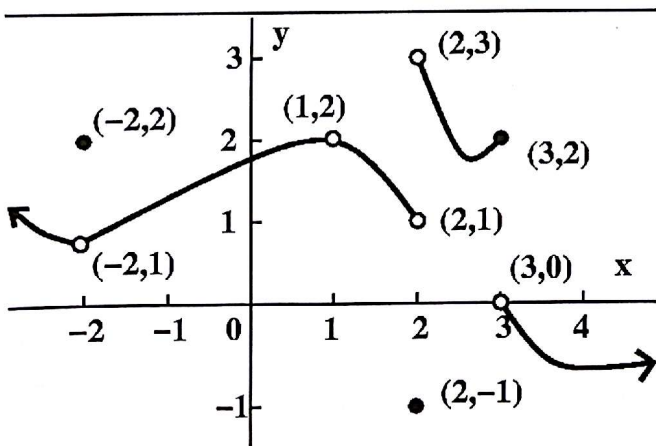


- (a) $\lim_{x \rightarrow 3^-} f(x) = 2$
- (b) $\lim_{x \rightarrow 3^+} f(x) = 1$
- (c) $\lim_{x \rightarrow 3} f(x) = \text{DNE}$
- (d) $f(3) = 1$



- (a) $\lim_{x \rightarrow 2^-} G(x) = \infty$
- (b) $\lim_{x \rightarrow 2^+} G(x) = -\infty$
- (c) $\lim_{x \rightarrow 2} G(x) = \text{DNE}$
- (d) $G(2) = 3$

Now you try:



$f(-2) = 2$	$\lim_{x \rightarrow 2^-} f(x) = 1$	$\lim_{x \rightarrow 2^+} f(x) = 1$	$\lim_{x \rightarrow 2} f(x) = 1$
$f(1) = \text{DNE}$	$\lim_{x \rightarrow 1^-} f(x) = 2$	$\lim_{x \rightarrow 1^+} f(x) = 2$	$\lim_{x \rightarrow 1} f(x) = 2$
$f(3) = 2$	$\lim_{x \rightarrow 3^-} f(x) = 2$	$\lim_{x \rightarrow 3^+} f(x) = 0$	$\lim_{x \rightarrow 3} f(x) = \text{DNE}$
$f(2) = -1$	$\lim_{x \rightarrow 2^-} f(x) = 1$	$\lim_{x \rightarrow 2^+} f(x) = 3$	$\lim_{x \rightarrow 2} f(x) = \text{DNE}$