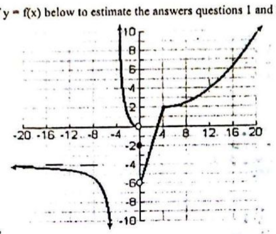
KEY

AB Calculus
Limits and Tangents Lab
Part 1: Non Calculator

1. Use the graph of y = f(x) below to estimate the answers questions 1 and 2.



a) $\lim_{x\to 0^+} f(x) =$ -6

b) $f(0) =$ -2

c) $\lim_{x\to 0^-} f(x) =$ 0

d) $\lim_{x\to 4^-} f(x) =$ 2

e) $\lim_{x\to -\infty} f(x) =$ -4

e) $\lim_{x\to 4} f(x) =$ DNE

2. Is f(x) continuous at the given x-value? Answer yes or no and use the definition of continuity to give reasons for your answer.

x = 0 No. $\lim_{x\to 0^-} f(x) = 0$ and $\lim_{x\to 0^+} f(x) = -6$ so $\lim_{x\to 0} f(x)$ DNE.

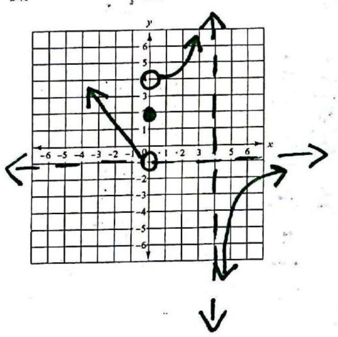
x = 4 yes $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 2$, $f(4) = 2$, $\lim_{x\to 4} f(x) = 2$ $\therefore \lim_{x\to 4} f(x) = f(4)$

3. Sketch a function that satisfies each of the following conditions:

$\lim_{x\to 0^+} f(x) = 4$ $f(0) = 2$ $\lim_{x\to 0^-} f(x) = -1$ $\lim_{x\to 4^-} f(x) = \infty$

$\lim_{x\to \infty} f(x) = -1$ $\lim_{x\to 4^+} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = \infty$



4. Evaluate the following limits. Classify any non-existent limits whenever possible. Show your work or explain your reasoning.

a. $\lim_{x\to 0} \frac{(3+x)^2 - 9}{x}$ 6

b. $\lim_{x\to -\infty} \left(\frac{\sqrt{2x^4 - 5x^2 + 10}}{1 - x^2} \right)$ $-\sqrt{2}$

c. $\lim_{x\to 2^-} \left| \frac{x+2}{x^2 - 4} \right|$ ∞

d. $\lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}$ $\frac{1}{2}$

e. $\lim_{x\to 0} \frac{\sin 4x}{5x}$ $\frac{4}{5}$

5. Find all the EXACT values of c so that $f(x)=\begin{cases} c & ,x<2 \\ 2\sqrt{cx}-1, & x\ge 2\end{cases}$ is continuous at x = 2.

Show all work.

$c=\dfrac{6\pm\sqrt{32}}{2}$

AB Calculus - Limits Lab
Part II: Calculator

6. Given $f(x)=\dfrac{2x^2-4x}{|2-x|}$, use the table provided below to compute the following limits.

Choose appropriate values for x.

	Left:				Right:			
x	1.9	1.99	1.999	1.9999	2.1	2.01	2.001	2.0001
y	−3.8	−3.98	−3.998	−3.9998	4.2	4.02	4.002	4.0002

a) $\lim\limits_{x\to 2^+} f(x)=$ 4 b) $\lim\limits_{x\to 2^-} f(x)=$ −4 $\lim\limits_{x\to 2} f(x)=$ DNE

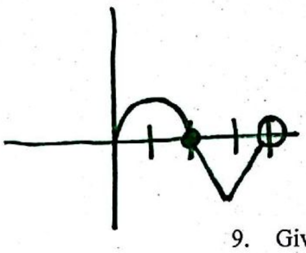
7. Prove that the function $f(x)=2x^3+x^2+2$ has at least one root on the interval [-2, -1]. Hint: You must use a theorem in your proof.

$f(-2)=-10$ and $f(-1)=1$
$\therefore$ there exists a value, c, in $[-2,-1]$ such that $f(c)=k$ is between −10 and 1 by the IVT

8. Let $g(x)=\begin{cases} 2x-x^2 & \text{if } 0\le x\le 2 \\ 2-x & \text{if } 2<x\le 3 \\ x-4 & \text{if } 3<x<4 \\ \pi & \text{if } x\ge 4\end{cases}$

a. Prove or disprove the function is continuous at the following points, using the formal definition of continuity.
   - d i. 0
   - c ii. 2
   - c iii. 3
   - d iv. 4

b. Sketch the graph of g



i) $\lim\limits_{x\to 0} g(x)=\text{DNE}$

ii) $\left.\begin{array}{l}\lim\limits_{x\to 2^-} g(x)=0 \\ \lim\limits_{x\to 2^+} g(x)=0\end{array}\right\}$ $\lim\limits_{x\to 2} g(x)=0$

$g(2)=0$

$\therefore \lim\limits_{x\to 2} g(x)=g(2)$

iii) $\left.\begin{array}{l}\lim\limits_{x\to 3^-} g(x)=-1 \\ \lim\limits_{x\to 3^+} g(x)=-1\end{array}\right\}$ $\lim\limits_{x\to 3} g(x)=-1$

$g(3)=-1$

$\therefore \lim\limits_{x\to 3} g(x)=g(3)$

iv) $\lim\limits_{x\to 4} g(x)=\text{DNE}$

9. Given the function: $f(x)=\dfrac{x^3-x}{x^2-6x+5}$

a. Find all asymptotes $x=5$

b. Find the roots of the function $(0,0),(-1,0)$

c. Find the domain of the function

c) $(-\infty,1)\cup(1,5)\cup(5,\infty)$

10. Find the values of a and b that make f continuous everywhere

$f(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2<x<3 \\ 2x-a+b & \text{if } x\ge 3\end{cases}$

$a=\frac{1}{2}$

$b=\frac{1}{2}$