L'Hopital's Rule:

1) $\lim_{x\to 0} x\csc x = \lim_{x\to 0} \frac{x}{\sin x}$

D

L'H: $\lim_{x\to 0} \frac{1}{\cos x} = 1$

2) $f'(x) = \cos x \quad g'(x) = 1 \quad f(0) = g(0) = 0$

B $\lim_{x\to 0} \frac{f(x)}{g(x)}$

L'H: $\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \lim_{x\to 0} \frac{\cos x}{1} = 1$

3) $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{2(1-\cos^2\theta)} = \lim_{\theta\to 0} \frac{\cancel{(1-\cos\theta)}}{2(1+\cos\theta)\cancel{(1-\cos\theta)}}$

C

$= \lim_{\theta\to 0} \frac{1}{2(1+\cos\theta)} = \frac{1}{4}$

4) $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$

C

L'H: $\lim_{h\to 0} \frac{\sqrt{(1+h)^5+8}}{1} = 3$

5) $\lim_{x\to \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)}$

D

L'H: $\lim_{x\to \frac{\pi}{4}} \frac{\cos\left(x-\frac{\pi}{4}\right)}{1} = 1$

[redacted]

6) $\lim\limits_{x \to 0} \frac{f(x)}{g(x)}$

C

L'H: $\lim\limits_{x \to 0} \frac{f'(x)}{g'(x)}$

7) $\lim\limits_{x \to 1} \frac{\int_1^x e^{t^2}}{x^2 - 1}$

C

L'H:

$\lim\limits_{x \to 1} \frac{e^{x^2}}{2x} = \frac{e}{2}$

9) $\lim\limits_{x \to \infty} (1 + 5e^x)^{1/x} = \infty^0$

C

$\ln y = \lim\limits_{x \to \infty} \frac{1}{x} \ln(1 + 5e^x)$

$\ln y = \lim\limits_{x \to \infty} \frac{\ln(1 + 5e^x)}{x}$

L'H: $\ln y = \lim\limits_{x \to \infty} \frac{1}{1 + 5e^x} \cdot 5e^x$

L'H: $\ln y = \lim\limits_{x \to \infty} \frac{5e^x}{5e^x} = 1$

$\ln y = 1$

$y = e$

8) $\lim\limits_{x \to 0} (1 + 2x)^{\csc x} = 1^\infty$

E

$y = \lim\limits_{x \to 0} (1 + 2x)^{\csc x}$

$\ln y = \lim\limits_{x \to 0} \csc x \cdot \ln(1 + 2x) = \infty \cdot 0$

$\ln y = \lim\limits_{x \to 0} \frac{\ln(1 + 2x)}{\sin x}$

L'H: $\ln y = \lim\limits_{x \to 0} \frac{\frac{2}{1 + 2x}}{\cos x}$

$\ln y = \frac{2}{1} = 2$

$y = e^2$

10) $\lim\limits_{x \to +\infty} \frac{x^k}{e^x} = 0$

A

big

$$\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$$

C

L'H: $$\lim_{x\to 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{e^0 + \sin 0 - 2}{-2} = \frac{-1}{-2} = \frac{1}{2}$$

12) $$\lim_{x\to 0} \frac{\sin x \cos x}{x}$$

C

L'H: $$\lim_{x\to 0} \frac{\sin x \cdot (-\sin x) + \cos x(\cos x)}{1}$$

$$= \lim_{x\to 0} -\sin^2 x + \cos^2 x = 1$$

13) $$\lim_{x\to 3} \frac{e^{x^2} - e^9}{x-3}$$

D

L'H: $$\lim_{x\to 3} e^{x^2} \cdot 2x = 6e^9$$

14) $$\lim_{x\to 1} \frac{\int_1^x \sin t \, dt}{x^2 - 1}$$

C

L'H: $$\lim_{x\to 1} \frac{\sin x}{2x} = \frac{\sin 1}{2}$$

15) $$\lim_{x\to 0} \frac{\cos x - e^x}{\ln(1+x)}$$

A

L'H: $$\lim_{x\to 0} \frac{-\sin x - e^x}{\frac{1}{1+x}} = -1$$

16) $\lim_{x \to \infty} [f(x)]^{1/x}$

B

$$y = \lim_{x \to \infty} [f(x)]^{1/x}$$

$$\ln y = \lim_{x \to \infty} \frac{1}{x} \ln f(x)$$

$$\ln y = \lim_{x \to \infty} \frac{\ln f(x)}{x}$$

$$\ln y \overset{L'H:}{=} \lim_{x \to \infty} \frac{1}{f(x)} \cdot f'(x)$$

$$\ln y = 0$$

$$y = 1$$

17) $\lim_{x \to 0} [e^x]^{f(x)}$

B

$$y = \lim_{x \to 0} [e^x]^{f(x)}$$

$$\ln y = \lim_{x \to 0} f(x) \cancel{\ln e}^x$$

$$\ln y = \lim_{x \to 0} x \cdot f(x)$$

$$\ln y = \lim_{x \to 0} \frac{f(x)}{\frac{1}{x}}$$

$$\ln y \overset{L'H:}{=} \lim_{x \to 0} \frac{f'(x)}{-\frac{1}{x^2}}$$ small } big

$$\ln y = \frac{4}{-\infty}$$

$$\ln y = 0$$

$$e^0 = 1$$

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Improper Integrals:

18) A

$$\int_2^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_2^b x^{-2}\,dx = -\frac{1}{x}\Big|_2^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{b}+\frac{1}{2}\right) = 0+\frac{1}{2} = \frac{1}{2}$$

19) E

$$\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx = \lim_{b\to\infty}\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$$

$u = 9-x^2$
$du = -2x\,dx$

$$\int u^{-1/3}\,du = \frac{3}{2}u^{2/3}$$

$$\lim_{b\to\infty}\left(\frac{3}{2}(9-x^2)^{2/3}\right)\Big|_4^\infty = \lim_{b\to\infty}\left(\frac{3}{2}\left(\sqrt[3]{9-x^2}\right)^2\right)\Big|_4^\infty$$

$= \text{divergent}$

20) C

$$\int_0^\infty x^2 e^{-x^3}\,dx = \lim_{b\to\infty}\int_0^b x^2 e^{-x^3}\,dx$$

$u = -x^3$
$du = -3x^2\,dx$
$-\frac{1}{3}du = x^2\,dx$

$$\lim_{b\to\infty} -\frac{1}{3}\int e^u\,du =$$

$$\lim_{b\to\infty} -\frac{1}{3}e^{-x^3}\Big|_0^b = \lim_{b\to\infty}\left(-\frac{1}{3}\cdot\frac{1}{e^{b^3}}+\frac{1}{3}\right)$$

$$= \frac{1}{3}$$

21) $\displaystyle\int_1^\infty \frac{1}{x^{2p}}\,dx$

C

$2p > 1 \quad p > \frac{1}{2}$

22) $\displaystyle\int_3^\infty e^{-2x}\,dx = \lim_{b\to\infty}\int_3^b e^{-2x}\,dx$

$u = -2x$
$du = -2dx$
$-\frac{1}{2}du = dx$

A

$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^{-2x}$

$\displaystyle\lim_{b\to\infty} \left.\frac{-1}{2e^{2x}}\right|_3^b = \lim_{b\to\infty}\left(\underbrace{\cancel{\frac{-1}{2e^{2b}}}}_{=0} + \frac{1}{2e^6}\right)$

$= \dfrac{1}{2e^6}$

23) $\displaystyle\int_1^\infty \frac{x^2}{(1+x^3)^2}\,dx$

D

$\displaystyle\lim_{b\to\infty}\int_1^b \frac{x^2}{(1+x^3)^2}\,dx =$

$u = 1+x^3$
$du = 3x^2dx$
$\frac{1}{3}du = x^2dx$

$\frac{1}{3}\int u^{-2}\,du = \dfrac{-1}{3u} = \dfrac{-1}{3(1+x^3)}$

$\displaystyle\lim_{b\to\infty} \left.\frac{-1}{3(1+x^3)}\right|_1^b = \lim_{b\to\infty}\left(\cancel{\frac{-1}{3(1+b^3)}} + \frac{1}{3(2)}\right)$

0

$= \dfrac{1}{6}$

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24) $\displaystyle\int_0^\infty \frac{3x^2}{(1+x^3)^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{3x^2}{(1+x^3)^2}\,dx$

C

$u=1+x^3$
$du=3x^2dx$

$\displaystyle\int u^{-2}du = -\frac{1}{u}$

$\displaystyle\lim_{b\to\infty} -\frac{1}{1+x^3}\Big|_0^b = \lim_{b\to\infty}\left(\cancel{\frac{-1}{1+b^3}}^{\,0} + 1\right)$
$= 1$

25) $\displaystyle 3\int_0^3 \frac{1}{x}\,dx = 3\cdot\lim_{t\to 0^+}\int_t^3 \frac{1}{x}\,dx$

E

$\displaystyle\lim_{t\to 0^+} 3\ln|x|\Big|_t^3 = 3\ln 3 - 3\ln t$
$\ln 27 - (3\cdot -\infty)$
divergent

26) $f(x) = -\ln x \quad 0 < x \leq 1$

a) $\displaystyle -\int_0^1 \ln x\,dx = \lim_{t\to 0^+} -\int_t^1 \ln x\,dx =$

$\therefore$ Finite

$\displaystyle\lim_{t\to 0^+} -(x\ln x - x)\Big|_t^1 =$
$-x\ln x + x$

$\displaystyle\lim_{t\to 0^+} (1-(-t\ln t + t))$

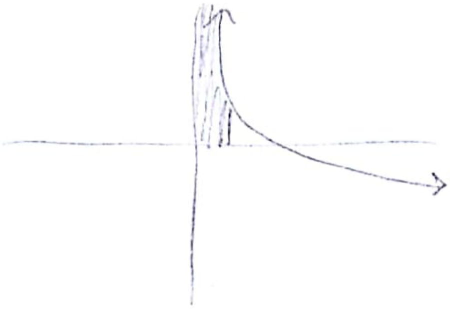
L'H: $\dfrac{\frac{1}{t}}{\frac{-1}{t^2}} = \dfrac{1}{t}\cdot\dfrac{-t^2}{1} = -t$

$\displaystyle\lim_{t\to 0^+} (1 + t\ln t - t) = \left(1 + \cancel{\frac{\ln t}{\frac{1}{t}}}^{\,0} - \cancel{t}\right)_0$
$= \boxed{1}$

-6) $y = -\ln x$

$-y = \ln x$

$x = e^{-y}$



$$\pi \int_0^{\infty} (e^{-y})^2 \, dy$$

$$\lim_{b\to\infty} \pi \int_0^{b} e^{-2y} \, dy =$$

$u = -2y$

$du = -2dy$

$-\frac{1}{2}du = dy$

$$-\frac{\pi}{2} \int e^u \, du = -\frac{\pi}{2} e^{-2y}$$

$$-\frac{\pi}{2} \lim_{b\to\infty} e^{-2y} \Big|_0^b = -\frac{\pi}{2} \lim_{b\to\infty} \left(\cancel{\frac{1}{e^{2b}}}^{\,0} - 1 \right)$$

$$-\frac{\pi}{2}(-1) = \frac{\pi}{2}$$

finite volume

27) $\displaystyle\int_0^{\infty} x e^{-x} \, dx$

	u	dv
$+$	x	e^{-x}
$-$	1	$-e^{-x}$
	0	e^{-x}

$$-xe^{-x} - e^{-x} \qquad \lim_{b\to\infty} \left(-xe^{-x} - e^{-x} \right) \Big|_0^b$$

$$\lim_{b\to\infty} \left(\cancel{-\frac{b}{e^b}}^{\,0} - \cancel{\frac{1}{e^b}}^{\,0} \right) - (-1) = 1$$

converges and $= 1$