Error Practice Day 2

1. Write the fourth degree Maclaurin polynomial for $f(x) = e^x$.
   a. Use your polynomial to approximate e^{-1}.
   b. Find a Lagrange error bound for the maximum error when $|x| \le 1$. Give three decimal places.
   c. Find an interval such $[a, b]$ such that $a \le e^{-1} \le b$
2. Suppose a function f is approximated with a fourth-degree Taylor polynomial about x = 1. If the maximum value of the fifth derivative between x = 1 and x = 3 is 0.01, that is $|f^5(x)| \le 0.01|$ then the maximum error incurred using this approximation to compute f(3) is
   a. 0.054 b. 0054 c. 0.2667 d. 0.02667 e. 0.00267
3. Let f be a function that has derivatives of all orders for all real numbers x Assume that

   $f(5)=6,\ f'(5)=8,\ f''(5)=30,\ f'''(5)=48,$ and $\left|f^{(4)}(x)\right| \le 75$ for all x in the interval $[5, 5.2]$.

   (a) Find the third-degree Taylor polynomial about $x = 5$ for $f(x)$.

   (b) Use your answer to part (a) to estimate the value of $f(5.2)$. What is the maximum possible error in making this estimate? Give three decimal places.

   (c) Find an interval $[a, b]$ such that $a \le f(5.2) \le b$. Give three decimal places.

   (d) Could $f(5.2)$ equal 8.254? Show why or why not.

4. Let f be the function given by $f(x) = \cos\left(2x + \frac{\pi}{6}\right)$ and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

   (a) Find $P(x)$.

   (b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{12{,}000}$.

5. Find the remainder when $\frac{1}{\sqrt{e}}$ is approximated by a third-degree MacLaurin polynomial.
   a. 0.0026
   b. 0.0208
   c. 0.2916
   d. 0.5833
   e. 1.6667

6. Using the formula for the error E, what is the maximum value of the error in approximating ln 1.2 with a Taylor polynomial of degree 3 centered at x =1?
   a. 0.000345
   b. 0.0004
   c. 0.00666
   d. 0.1813
   e. 0.1827

switch to notes

2004 BC6 parts (a) and (c)

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Let f be the function given by $f(x)=\sin\left(5x+\frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree taylor polynomial for f about $x=0$.

(a) Find $P(x)$.

...

(b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right)-P\left(\frac{1}{10}\right)\right|<\frac{1}{100}$.

7. The function f has derivatives of all orders for all real numbers x. Assume $f(2)=-3$, $f'(2)=5$, $f''(2)=3$, and $f'''(2)=-8$.

(a) Write the third-degree Taylor polynomial for f about $x=2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)|\le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5)\ne -5$.

Error Practice Day 2:

1. $P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

a) $P_4(-1) = .375$

b) $E \le \left| \frac{e^1(-1-0)^5}{5!} \right|$

$-1 \le z \le 1$ $\downarrow$ e^1 maximizes

$E \le .0227$

c) $.375 \pm .0227$

$.3523 \le e^{-1} \le .3977$

2. E

$E \le \left| \frac{.01(3-1)^5}{5!} \right|$

$E \le .00267$

3. a) $6 + 8(x-5) + \frac{30(x-5)^2}{2!} + \frac{48(x-5)^3}{3!}$

b) $f(5.2) \approx 8.264$

$\text{Error} \le \left| \frac{75(5.2-5)^4}{4!} \right|$

$\text{Error} \le .005$

c) $8.264 \pm .005$

$8.259 \le f(5.2) \le 8.269$

d) No, it's not in interval

④ a) $f(x) = \cos(2x + \frac{\pi}{6})$ $f(0) = \frac{\sqrt{3}}{2}$

$f'(x) = -2\sin(2x + \frac{\pi}{6})$ $f'(0) = -1$

$f''(x) = -4\cos(2x + \frac{\pi}{6})$ $f''(0) = -2\sqrt{3}$

$f'''(x) = 8\sin(2x + \frac{\pi}{6})$ $f'''(0) = 4$

$$P_3(x) = \frac{\sqrt{3}}{2} - x - \frac{2\sqrt{3}x^2}{2!} + \frac{4x^3}{3!}$$

b) $f^4(x) = 16\cos(2x + \frac{\pi}{6})$

max $= 16$

$$\text{Error} \leq \left| \frac{16\left(\frac{1}{10}\right)^4}{4!} \right|$$

$$6.6 \times 10^{-5} < \frac{1}{12{,}000} = 8.3 \times 10^{-5}$$

⑤ $e^{-1/2}$, 3rd degree MacLaurin

(A) $$E \leq \left| \frac{1 \cdot \left(-\frac{1}{2}\right)^4}{4!} \right|$$

$-\frac{1}{2} \leq z \leq 0$

$\downarrow$

$1 = e^0 = \text{max}$

$E \leq .0026$

⑥ 2004 BC6

$f(x) = \sin(5x + \frac{\pi}{4})$ $\quad f(0) = \frac{\sqrt{2}}{2}$

$f'(x) = 5\cos(5x + \frac{\pi}{4})$ $\quad f'(0) = \frac{5\sqrt{2}}{2}$

$f''(x) = -25\sin(5x + \frac{\pi}{4})$ $\quad f''(0) = -\frac{25\sqrt{2}}{2}$

$f'''(x) = -125\cos(5x + \frac{\pi}{4})$ $\quad f'''(0) = -\frac{125\sqrt{2}}{2}$

a) $P(x) = \frac{\sqrt{2}}{2} + \frac{5\sqrt{2}x}{2} - \frac{25\sqrt{2}x^2}{2 \cdot 2!} - \frac{125\sqrt{2}x^3}{2 \cdot 3!}$

b) $R_3(x) < \left| \frac{f^4(z) \cdot (x-c)^4}{4!} \right|$

$f^4(x) = 625 \sin(5x + \frac{\pi}{4})$

$R_3(x) < \left| \frac{625 (\frac{1}{10})^4}{4!} \right|$

$\frac{1}{384} < \frac{1}{100}$

7 a) $P_3(x) = -3 + 5(x-2) + \frac{3(x-2)^2}{2!} - \frac{8(x-2)^3}{3!}$

$P_3(1.5) = -4.958$

b) $\max f^4(x) = 3 \quad [1.5, 2]$

$R_3(x) < \left| \frac{3(1.5-2)^4}{4!} \right|$

$R_3(x) < .0078125$

$f(1.5) = -4.958 \pm .0078125$

$-4.966 < f(1.5) < -4.950$

-5 does not fall within interval