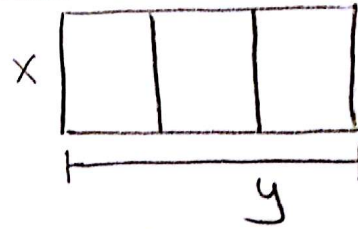


AP Calculus AB  
Introduction to Optimization

Name: Key

1. A farmer has 120 feet of fencing with which to enclose three adjacent rectangular pens. Draw a diagram and find the dimensions that will produce the maximum area and find the maximum area.



Method 1:

a) Write an equation for the total amount of fencing used.

$$120 = 4x + 2y$$

b) Write an equation for the total area of the pens.

$$A = xy$$

c) Solve your equation in part a) for y and substitute into your equation from b).

$$2y = -4x + 120$$

$$y = -2x + 60$$

d) Using your calculator, graph your equation in part c). What value for x produces a maximum point?

$$x = 15 \text{ ft}$$

e) Using your x-value from part d), solve for y and the maximum area.

$$y = 30 \text{ ft}$$

$$A = 450 \text{ ft}^2$$

$$A = x(-2x + 60)$$

$$A = -2x^2 + 60x$$

Method 2: Calculus!

$$A = -2x^2 + 60x$$

$$\frac{dA}{dx} = -4x + 60 = 0$$

$$x = 15 \text{ ft}$$

$$y = 30 \text{ ft} \quad A = 450 \text{ ft}^2$$

2. Two positive numbers have a sum of 10. Find the two numbers so that the product of three times the first and two times the second is a maximum.

Method 1:

a) Write two equations to illustrate the problem.

$$x + y = 10$$

$$3x \cdot 2y = M$$

$$y = 10 - x$$

b) Solve one equation in part a) for one of the variables. Substitute the equation into the other equation from part a).

c) Using your calculator, graph the new equation from part b) to determine the values needed to answer the question.

$$3x \cdot 2(10 - x) = M$$

$$3x \cdot (20 - 2x) = M$$

$$60x - 6x^2 = M$$

$$x = 5$$

$$y = 5$$

Method 2: Calculus!

$$60x - 6x^2 = M$$

$$60 - 12x = \frac{dM}{dx}$$

$$x = 5$$

$$y = 5$$