Review Part II:

(1a) $\int_0^\infty e^{-x}dx$

$\lim_{K\to\infty} \int_0^K e^{-x}dx$

$\lim_{K\to\infty} -e^{-x}\Big|_0^K$

$\lim_{K\to\infty}\left(\cancel{-\frac{1}{e^K}}_{0} - \left(\frac{-1}{e^0}\right)\right)$

$\boxed{1}$

Semi-circles: $A = \frac{\pi\left(\frac{1}{2}e^{-x}\right)^2}{2}$

$d = e^{-x}$

$r = \frac{1}{2}e^{-x}$ $\quad A = \frac{1}{8}\pi e^{-2x}$

$\lim_{K\to\infty} \frac{1}{8}\pi\int_0^K e^{-2x}dx$

$\lim_{K\to\infty} \frac{1}{8}\pi\left(-\frac{1}{2}e^{-2x}\Big|_0^K\right)$

$\lim_{K\to\infty} \frac{1}{8}\pi\left(\cancel{-\frac{1}{2e^{-2K}}}_{0} - \left(\frac{-1}{2e^0}\right)\right)$

$\frac{1}{8}\pi \cdot \frac{1}{2} = \boxed{\frac{1}{16}\pi}$

(1b) $\lim_{K\to\infty} \pi\int_0^K (e^{-x})^2 dx$

$\lim_{K\to\infty} \pi\left(-\frac{1}{2}e^{-2x}\right)\Big|_0^K$

$\lim_{K\to\infty} \pi\left(\cancel{\frac{-1}{2e^{2K}}}_{0} - \left(\frac{-1}{2e^0}\right)\right)$

$\pi\left(\frac{1}{2}\right) = \boxed{\frac{1}{2}\pi}$

(1c) Squares: $A = (e^{-x})^2$

$\lim_{K\to\infty} \int_0^K (e^{-x})^2 dx$

$\lim_{K\to\infty} -\frac{1}{2}e^{-2x}\Big|_0^K$

$\lim_{K\to\infty}\left(-\frac{1}{2e^{2K}} - \left(\frac{-1}{2e^0}\right)\right)$

$= \boxed{\frac{1}{2}}$

(2) $y = x\sin x - 3\cos x$

$y' = x\cdot\cos x + \sin x + 3\sin x$

$y'' = x\cdot(-\sin x) + \cos x + \cos x + 3\cos x$

$y''\ -x\sin x + 5\cos x$

(3) $y = \sqrt{4+\sqrt{3x}}$

$y = \left(4 + (3x)^{1/2}\right)^{1/2}$

$y' = \frac{1}{2}\left(4 + (3x)^{1/2}\right)^{-1/2} \cdot \left(\frac{1}{2}(3x)^{-1/2} \cdot 3\right)$

$y' = \frac{3}{4\sqrt{4+\sqrt{3x}}\cdot\sqrt{3x}}$

④ $x^2-4x+y^2+3=0$ $\quad y=Kx$

$y^2=-x^2+4x-3$ $\quad y'=K$

$y=\pm\sqrt{-x^2+4x-3}$

$y'=\frac{1}{2}(-x^2+4x-3)^{-1/2}\cdot(-2x+4)$

$y'=\frac{-2x+4}{2\sqrt{-x^2+4x-3}}$

$Kx=\sqrt{-x^2+4x-3}$ $\quad K=\frac{-2x+4}{2\sqrt{-x^2+4x-3}}$

$K=\frac{\sqrt{-x^2+4x-3}}{x}$

$\frac{\sqrt{-x^2+4x-3}}{x}=\frac{-2x+4}{2\sqrt{-x^2+4x-3}}$

$-2x^2+4x=2(-x^2+4x-3)$

$-2x^2+4x=-2x^2+8x-6$

$4x=8x-6$

$-4x=-6$

$x=\frac{3}{2}$

$K=\frac{\sqrt{-\left(\frac{3}{2}\right)^2+4\left(\frac{3}{2}\right)-3}}{\frac{3}{2}}$

$K=\frac{\sqrt{\frac{-9}{4}+\frac{12}{2}-3}}{\frac{3}{2}}$

$K=\frac{\sqrt{\frac{3}{4}}}{\frac{3}{2}}=\frac{\sqrt{3}}{2}\cdot\frac{2}{3}=\pm\frac{\sqrt{3}}{3}$

$y=\pm\frac{\sqrt{3}}{3}x$

⑤ $r=10$

$h=24$

$\frac{dV}{dt}=20\ ft^3/min$

$\frac{dh}{dt}=?$ when $h=16$

$\frac{r}{h}=\frac{10}{24}$ $\quad 24r=10h$

$r=\frac{10h}{24}=\frac{5}{12}h$

$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi\left(\frac{5}{12}h\right)^2\cdot h$

$V=\frac{1}{3}\pi\cdot\frac{25}{144}h^2\cdot h$

$V=\frac{25}{432}\pi h^3$

$\frac{dV}{dt}=\frac{25}{144}\pi h^2\frac{dh}{dt}$

$20=\frac{25}{144}\pi(16)^2\left(\frac{dh}{dt}\right)$

$\frac{9}{20\pi}\ ft/min=\frac{dh}{dt}$

⑥

a) $\int xe^{-12x}dx$

	u	dv
$+$	x	e^{-12x}
$-$	1	$-\frac{1}{12}e^{-12x}$
	0	$\frac{1}{144}e^{-12x}$

$-\frac{1}{12}xe^{-12x}-\frac{1}{144}e^{-12x}+C$

b) $\int\frac{4x+4}{(x-5)(x+3)}dx$

$\frac{4x+4}{(x-5)(x+3)}=\frac{A}{x-5}+\frac{B}{x+3}$

$4x+4=A(x+3)+B(x-5)$

$x=-3:\ -8=-8B\quad B=1$

$x=5:\ 24=8A\quad A=3$

$3\int\frac{1}{x-5}dx+\int\frac{1}{x+3}dx$

$3\ln|x-5|+\ln|x+3|+C$

$\ln\left|(x-5)^3(x+3)\right|+C$

c) $\int e^{-x}\sin x\,dx$ $\quad u=\sin x \quad v=-e^{-x}$
$du=\cos x\,dx \quad dv=e^{-x}dx$

$-e^{-x}\cdot\sin x+\int e^{-x}\cdot\cos x\,dx$ $\quad u=\cos x \quad v=-e^{-x}$
$du=-\sin x\,dx \quad dv=e^{-x}dx$

$-e^{-x}\sin x+\left(-e^{-x}\cos x-\int e^{-x}\sin x\,dx\right)$

$-e^{-x}\sin x-e^{-x}\cos x-\int e^{-x}\sin x\,dx=\int e^{-x}\sin x\,dx$

$C+\dfrac{-e^{-x}\sin x-e^{-x}\cos x}{2}=\int e^{-x}\sin x\,dx$

d) $\int x(x+1)^{1/4}dx$ $\quad u=x+1 \quad x=u-1$
$du=dx$

$\int(u-1)(u)^{1/4}du=\int\left(u^{5/4}-u^{1/4}\right)du$

$=\frac{4}{9}u^{9/4}-\frac{4}{5}u^{5/4}+C$

$=\frac{4}{9}(x+1)^{9/4}-\frac{4}{5}(x+1)^{5/4}+C$

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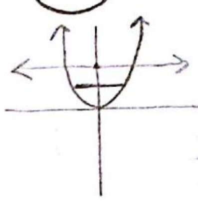

$\int_0^3\left(2\sqrt{y}\right)^2dy$

$4\int_0^3 y\,dy=\frac{4y^2}{2}\Big|_0^3=2y^2\Big|_0^3$
$=18$

e) $\int(3x+9)^{10}dx$ $\quad u=3x+9$
$du=3dx$
$\frac{1}{3}du=dx$

$\frac{1}{3}\int u^{10}du=\frac{1}{3}\cdot\frac{1}{11}u^{11}+C$

$\dfrac{(3x+9)^{11}}{33}+C$

f) $\int\sec^2x\left(4\tan^3x-3\tan^2x\right)dx$ $\quad u=\tan x$
$du=\sec^2x\,dx$

$4\int\sec^2x\tan^3x\,dx-3\int\sec^2x\tan^2x\,dx$

$4\int u^3du-3\int u^2du=$

$\frac{4u^4}{4}-\frac{3u^3}{3}=u^4-u^3=\tan^4x-\tan^3x+C$

(8) a) $\int_0^\infty (x+1)^{-1}dx$

$\lim_{b\to\infty} \int_0^b (x+1)^{-1}dx$ $\quad u=x+1$, $du=dx$

$\lim_{b\to\infty} \ln|x+1| \Big|_0^b$ $\quad \int u^{-1}du = \ln|u|$

$\lim_{b\to\infty} (\ln|b+1| - \cancel{\ln 1}^{\,0})$ ∴ divergent

infinite

b) $\pi\int_0^\infty ((x+1)^{-1})^2 dx$ $\quad u=x+1$, $du=dx$

$\lim_{b\to\infty} \pi \int_0^b (x+1)^{-2}dx$ $\quad \int u^{-2}du$, $-\frac{1}{u}$

$\lim_{b\to\infty} \pi\left(\frac{-1}{x+1}\Big|_0^b\right)$

$\lim_{b\to\infty} \pi\left(\cancel{\frac{-1}{b+1}}^{\,0} - \left(\frac{-1}{0+1}\right)\right)$

$= \pi$

c) $y = (x+1)^{-1} = \frac{1}{x+1}$

$(x+1)\cdot y = 1$

$x+1 = \frac{1}{y}$

$x = \frac{1}{y} - 1 = \frac{1-y}{y}$

$\pi\int_0^1 \left(\frac{1-y}{y}\right)^2 dy = \pi\int_0^1 \frac{1-2y+y^2}{y^2}dy$

$\lim_{a\to 0^+} \pi\int_a^1 (y^{-2} - \frac{2}{y} + 1)dy$

$\lim_{a\to 0^+} \pi\left(-\frac{1}{y} - 2\ln y + y \Big|_a^1\right)$

$\lim_{a\to 0^+} \pi\left[\left(-1 - 2\ln 1^{\,0} + 1\right) - \left(-\frac{1}{a} - 2\ln a + a\right)\right]$

$\pi\left(\frac{1}{a} + 2\ln a - a\right) = \pi(\infty - \infty)$

$\to \pi\left(\frac{1+2a\ln a}{a}\right)$

$\pi\left(\frac{1+2\left(\frac{\ln a}{\frac{1}{a}}\right)}{a}\right)$ ∴ div infinite

$\pi\left(\frac{1+2(-1)}{\text{small}}\right) = -\infty$

(9) $\lim_{x\to 1} \frac{e^x - e}{\ln x}$ $\quad \frac{0}{0}$

L'Hopital:

$\lim_{x\to 1} \frac{e^x}{\frac{1}{x}} = e$

(10) $\int x^2\sqrt{x+1}\,dx$ $\quad u=x+1$, $du=dx$, $x=u-1$, $x^2=(u-1)^2$

A

$\int ((u-1)^2(u)^{1/2})du$

$\int (u^2-2u+1)(u^{1/2})du$

$\int (u^{5/2} - 2u^{3/2} + u^{1/2})du$

$\frac{2}{7}u^{7/2} - 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$\frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

(11) $\pi\int_0^\infty \left(\frac{5}{x+1}\right)^2 dx$

D

$\lim_{b\to\infty} 25\pi\int_0^b \frac{1}{(x+1)^2}dx$ $\quad u=x+1$, $du=dx$, $\int u^{-2}du$, $-\frac{1}{u}$

$\lim_{b\to\infty} 25\pi\left(\frac{-1}{x+1}\Big|_0^b\right)$

$\lim_{b\to\infty} 25\pi\left(\cancel{\frac{-1}{b+1}}^{\,0} - \left(\frac{-1}{0+1}\right)\right)$

$= 25\pi$

(13) $y = \dfrac{3x+4}{4x-3}$

A

$$y' = \frac{(4x-3)(3)-(3x+4)(4)}{(4x-3)^2}$$

$$y'\Big|_{(1,7)} = \frac{1(3)-7(4)}{1}$$

$$y'(1) = -25$$

$$y-7 = -25(x-1)$$

$$y-7 = -25x+25$$

$$y+25x = 32$$

(15) $y' = \cos(2\pi t)$

$y(0)=1$

t	y	y'	
0	1	1	
0.2	1.2	.309	$y=1(.2)+1$
0.4	1.262	-.809	$y=.309(.2)+1.2$
0.6	1.1	-.809	$y=-.809(.2)+1.262$
0.8	.938	.309	$y=-.809(.2)+1.1$
1	1		$y=.309(.2)+.938$

(16) $y' = x+y$

$y(0)=1$

x	y	dy/dx	
0	1	1	
0.2	1.2	1.4	$y=1(.2)+1$
0.4	1.48	1.88	$y=1.4(.2)+1.2$
0.6	1.856	2.456	$y=1.88(.2)+1.48$
0.8	2.3472	3.1472	$y=2.456(.2)+1.856$
1	$\downarrow$		$y=3.1472(.2)+2.3472$

$$y(1) \approx 2.98$$

(17) $\displaystyle\int_2^4 \frac{1}{(x-3)^2}\,dx$ discont at $x=3$

$$\int_2^3 \frac{1}{(x-3)^2}\,dx + \int_3^4 \frac{1}{(x-3)^2}\,dx$$

$$\lim_{b\to 3^-}\int_2^b \frac{1}{(x-3)^2}\,dx + \lim_{a\to 3^+}\int_a^4 \frac{1}{(x-3)^2}\,dx$$

$$\lim_{b\to 3^-}\left(\frac{-1}{x-3}\Big|_2^b\right) + \lim_{a\to 3^+}\left(\frac{-1}{x-3}\Big|_a^4\right)$$

$u=x-3$

$du=dx$

$\displaystyle\int \frac{1}{u^2}\,du = \frac{-1}{u}$

$\dfrac{-1}{x-3}$

$$\lim_{b\to 3^-}\left(\frac{-1}{b-3} - \left(\frac{-1}{2-3}\right)\right)$$

$\downarrow$

$-\infty - 1$ $\therefore$ divergent

(18) $\displaystyle\int_2^4 \frac{1}{(x-3)^{2/3}}\,dx$ discont at $x=3$

$$\int_2^3 \frac{1}{(x-3)^{2/3}}\,dx + \int_3^4 \frac{1}{(x-3)^{2/3}}\,dx$$

$\displaystyle\int (x-3)^{-2/3}\,dx$

$u=x-3$

$\displaystyle\int u^{-2/3}\,du$ $du=dx$

$3u^{1/3}$

$$\lim_{b\to 3^-}\int_2^b \frac{1}{(x-3)^{2/3}}\,dx + \lim_{a\to 3^+}\int_a^4 \frac{1}{(x-3)^{2/3}}\,dx$$

$$\lim_{b\to 3^-}\left(3(x-3)^{1/3}\Big|_2^b\right) + \lim_{a\to 3^+}\left(3(x-3)^{1/3}\Big|_a^4\right)$$

$$\lim_{b\to 3^-}\left(3(b-3)^{1/3} - 3(2-3)^{1/3}\right) + \lim_{a\to 3^+}\left(3(4-3)^{1/3} - 3(a-3)^{1/3}\right)$$

$$(0+3) + (3-0) = 6$$

(19) $\displaystyle\pi\int_1^\infty \left(\frac{1}{x}\right)^2 dx$

$$\lim_{b\to\infty}\pi\int_1^b \frac{1}{x^2}\,dx$$

$$\lim_{b\to\infty}\pi\left(-\frac{1}{x}\Big|_1^b\right)$$

$$\lim_{b\to\infty}\pi\left(\cancel{\frac{-1}{b}} - \left(-\frac{1}{1}\right)\right)$$

0

$$=\pi$$